

THE MANGA GUIDE™ TO

COMICS  
INSIDE!

# RELATIVITY

HIDEO NITTA  
MASAFUMI YAMAMOTO  
KEITA TAKATSU  
TREND-PRO CO., LTD.





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—DAILY TECH ON *THE MANGA GUIDE TO CALCULUS*

"The art is fantastic, and the teaching method is both fun and educational."

—ACTIVE ANIME ON *THE MANGA GUIDE TO PHYSICS*

"An awfully fun, highly educational read."

—FRAZZLEDDAD ON *THE MANGA GUIDE TO PHYSICS*

"Makes it possible for a 10-year-old to develop a decent working knowledge of a subject that sends most college students running for the hills."

—SKEPTICBLOG ON *THE MANGA GUIDE TO MOLECULAR BIOLOGY*

"This book is by far the best book I have read on the subject. I think this book absolutely rocks and recommend it to anyone working with or just interested in databases."

—GEEK AT LARGE ON *THE MANGA GUIDE TO DATABASES*

"The book purposefully departs from a traditional physics textbook and it does it very well."

—DR. MARINA MILNER-BOLOTIN, RYERSON UNIVERSITY ON *THE MANGA GUIDE TO PHYSICS*

"Kids would be, I think, much more likely to actually pick this up and find out if they are interested in statistics as opposed to a regular textbook."

—GEEK BOOK ON *THE MANGA GUIDE TO STATISTICS*

"*The Manga Guide to Statistics* offers a visualization of statistics that can't be found in any mere textbook."

—ANIME 3000

"A great introduction for readers of any age, and an exemplar of technical communication."

—LINUX USERS OF VICTORIA ON *THE MANGA GUIDE TO ELECTRICITY*

THE MANGA GUIDE™ TO RELATIVITY



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**RELATIVITY**

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KEITA TAKATSU  
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# PREFACE

Welcome to the world of relativity!

Everyone wonders what relativity is all about. Because the theory of relativity predicts phenomena that seem unbelievable in our everyday lives (such as the slowing of time and the contraction of the length of an object), it can seem like mysterious magic.

Despite its surprising, counterintuitive predictions, Einstein's theory of relativity has been confirmed many times over with countless experiments by modern physicists. Relativity and the equally unintuitive quantum mechanics are indispensable tools for understanding the physical world.

In Newton's time, when physicists considered velocities much smaller than the speed of light, it was not a problem to think that the measurement of motion, that is, space and time, were independent, permanent, and indestructible absolutes. However, by the end of the 19th century, precise measurements of the speed of light combined with developments in the study of electromagnetism had set the stage for the discovery of relativity. As a result, time and space, which had always been considered to be independent and absolute, had to be reconsidered.

That's when Einstein arrived on the scene. Einstein proposed that time and space were in fact relative. He discarded the idea that space and time were absolute and considered that they vary together, so that the speed of light is always constant.

This radical insight created a controversy just as Galileo's claim that Earth orbited the Sun (and not vice versa) shocked his peers. However, once we ventured into space, it was obvious that Earth was indeed moving.

In a similar way, relativity has given us a more accurate understanding of concepts regarding the space-time in which we are living. In other words, relativity is the result of asking what is *actually* happening in our world rather than saying our world *should be* a particular way.

Although this preface may seem a little difficult, I hope you will enjoy the mysteries of relativity in a manga world together with Minagi and his teacher, Miss Uruga. Finally, I'd like to express my deep gratitude to everyone in the development bureau at Ohmsha; re\_akino, who toiled over the scenario; and Mr. Keita Takatsu, who converted it into such an interesting manga.

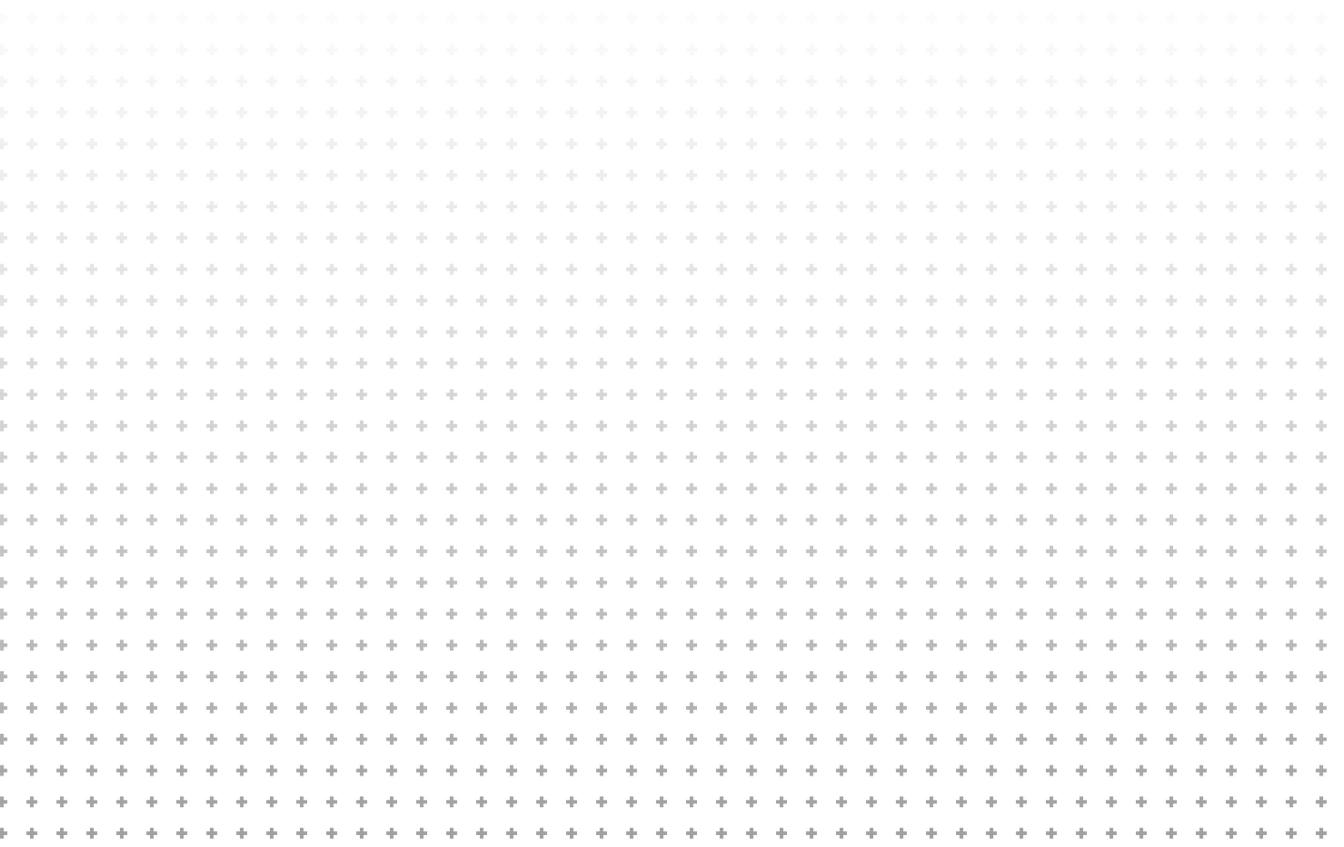
Well, then. Let's jump into the world of relativity.

**MASAFUMI YAMAMOTO**  
JUNE 2009





# OUTRAGEOUS CLOSING CEREMONY



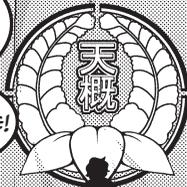
TAIGAI ACADEMY,  
THE LAST DAY  
OF SCHOOL  
BEFORE  
SUMMER

概学園

THE SCHOOL IS  
BRIMMING WITH ANTICI-  
PATION OF SWIMMING AT  
THE BEACH...CAMPING...  
AND SUMMER  
LOVE.

I REALIZE THAT  
YOUR SUMMER  
VACATION IS ABOUT  
TO BEGIN...

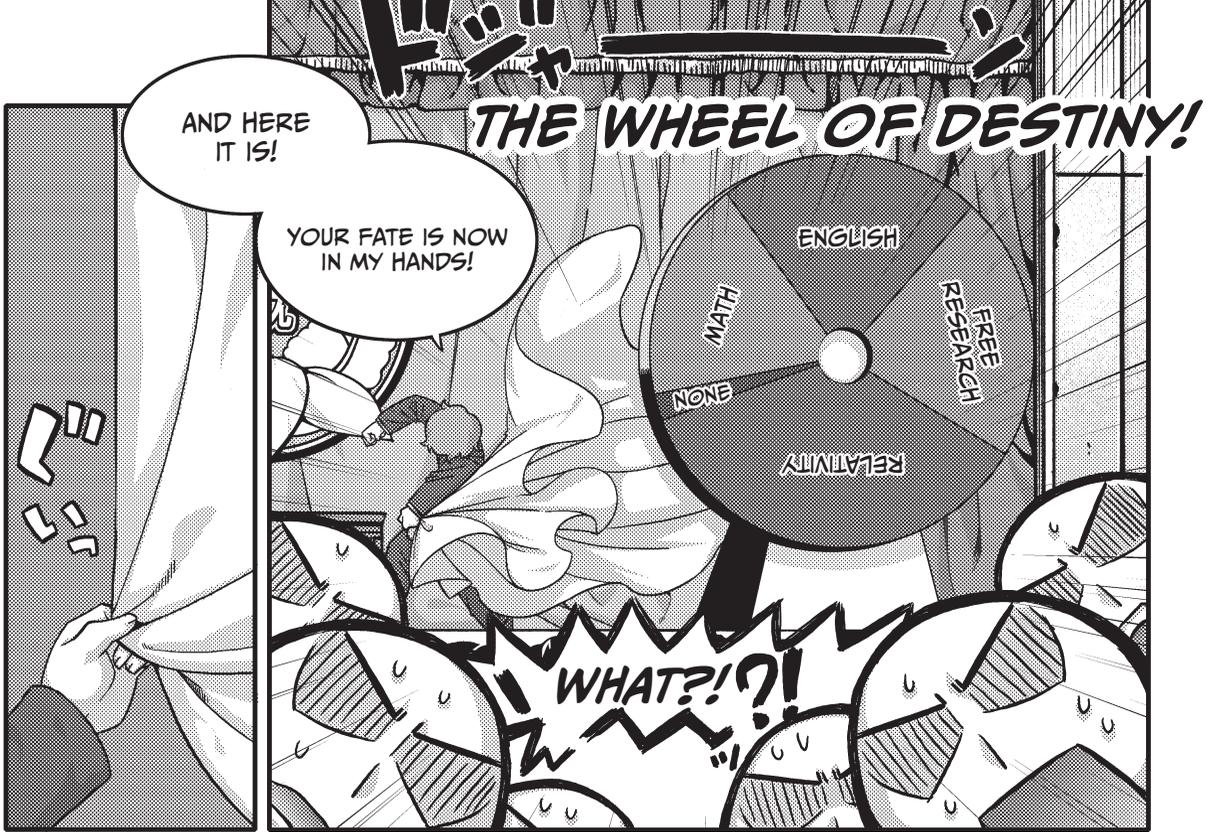
THEREFORE!



I'M GIVING THIS YEAR'S  
JUNIOR CLASS A SPECIAL  
PRESENT SO THAT YOU  
CAN ENJOY YOUR SUMMER  
BREAK EVEN MORE.

HEADMASTER  
RASE IYAGA

WHAT'S HE TRYING TO  
PULL? THE HEADMASTER  
IS SUCH A CREEP!

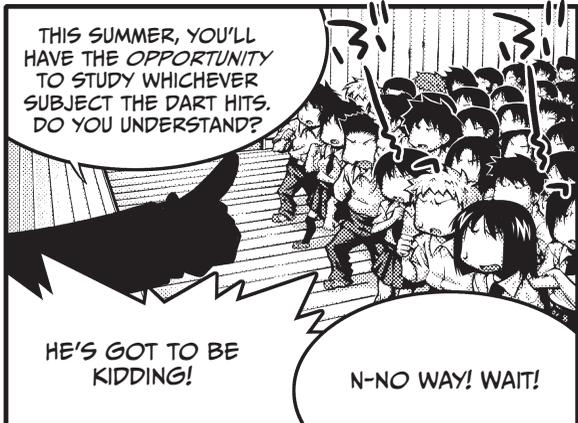


AND HERE IT IS!

YOUR FATE IS NOW IN MY HANDS!

# THE WHEEL OF DESTINY!

WHAT?!?!



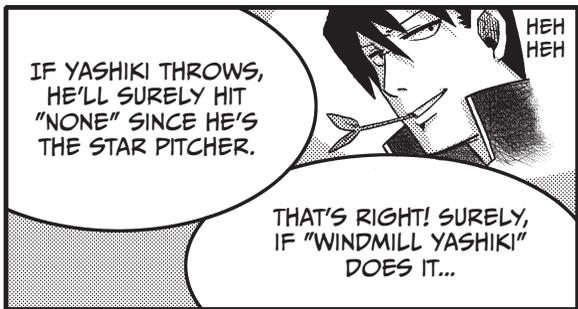
THIS SUMMER, YOU'LL HAVE THE OPPORTUNITY TO STUDY WHICHEVER SUBJECT THE DART HITS. DO YOU UNDERSTAND?

HE'S GOT TO BE KIDDING!

N-NO WAY! WAIT!



OKAY. HERE WE GO!



IF YASHIKI THROWS, HE'LL SURELY HIT "NONE" SINCE HE'S THE STAR PITCHER.

THAT'S RIGHT! SURELY, IF "WINDMILL YASHIKI" DOES IT...

OH NO! HEADMASTER IYAGA WILL BE DOING IT!

THAT'S DEVIIOUS,  
HEADMASTER!  
IT'S TYRANNY!

AND WHAT THE HECK  
IS RELATIVITY?

.....!!

RELATIVITY

WHAT THE...?!

THE WORST ONE!  
WHAT IS THAT?!

WHAT'S  
THIS...

THE VICE  
PRINCIPAL IS  
HERE!

WOOF!  
WOOF!

IS HE MESSING  
WITH US?!

VICE PRINCIPAL  
KOROMARU

WHAT?  
VICE PRINCIPAL, YOU'VE  
ALREADY TOLD THEM  
ABOUT THIS, RIGHT?

WE CAN'T UNDERSTAND  
THE VICE PRINCIPAL, BUT  
HE LOOKS ANGRY AT THE  
HEADMASTER!

WHY IS A DOG THE VICE  
PRINCIPAL AROUND  
HERE, ANYWAY?

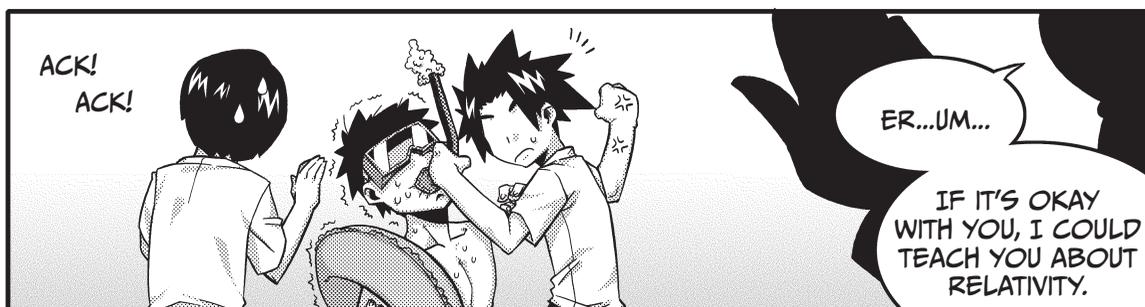
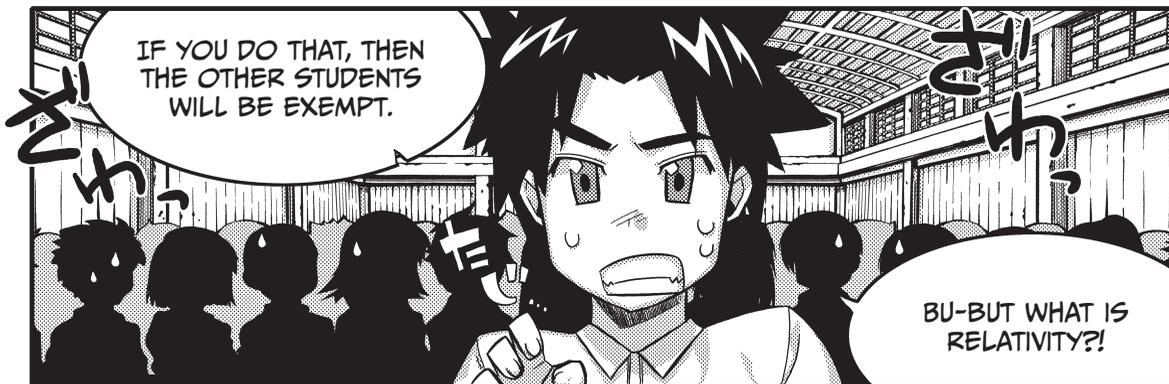
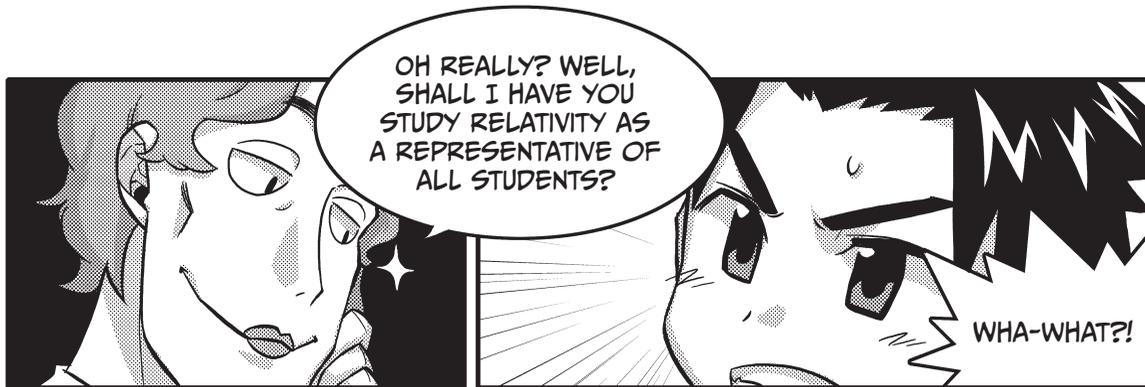
HEY, YOU CAN'T JUST  
SPRING THIS ON US!

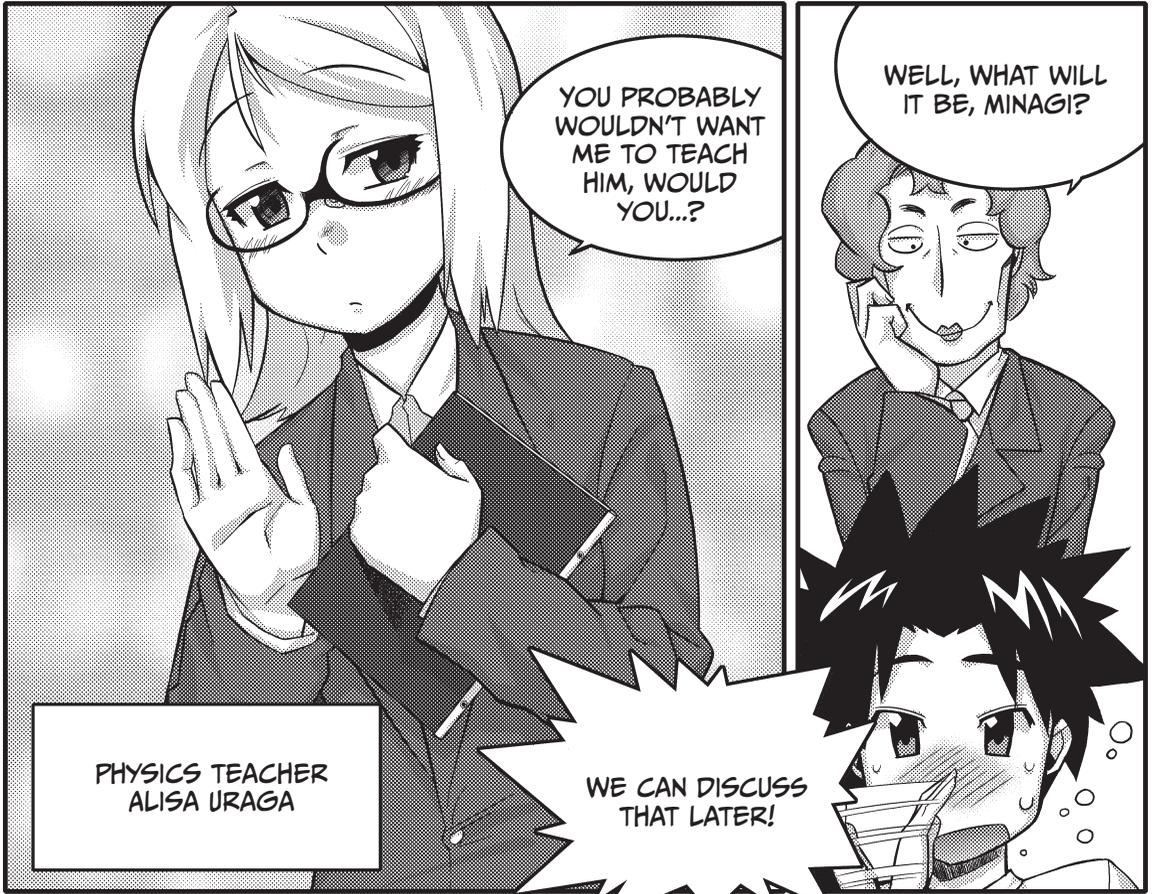
GOOD GRACIOUS! IT'S  
YOU, STUDENT BODY  
PRESIDENT SO-AND-SO.

THE WHEEL OF DESTINY  
IS A FARCE! YOU'RE JUST  
TORMENTING US.

EVERYONE HAS MADE  
THEIR PLANS FOR SUMMER  
VACATION ALREADY!

STUDENT BODY  
PRESIDENT  
RUKA MINAGI





YOU PROBABLY WOULDN'T WANT ME TO TEACH HIM, WOULD YOU...?

WELL, WHAT WILL IT BE, MINAGI?

PHYSICS TEACHER ALISA URAGA

WE CAN DISCUSS THAT LATER!



COME ON MAN! SAVE US!

WE BEG YOU, MAN.

DO IT!

BLEH.



OKAY, I ACCEPT YOUR CHALLENGE!!!

ALL RIGHT THEN. WHEN SUMMER VACATION IS OVER, SUBMIT A REPORT ON RELATIVITY.

OKAY!

I DON'T CARE IF MISS URAGA TEACHES YOU, BUT YOU HAVE TO WRITE THE REPORT YOURSELF!

AND IF YOU CAN'T DO IT...



IF I CAN'T DO IT...?

YOU'LL SPEND YOUR SENIOR YEAR AS MY...  
PERSONAL SECRETARY!  
NO FREAKING WAY!

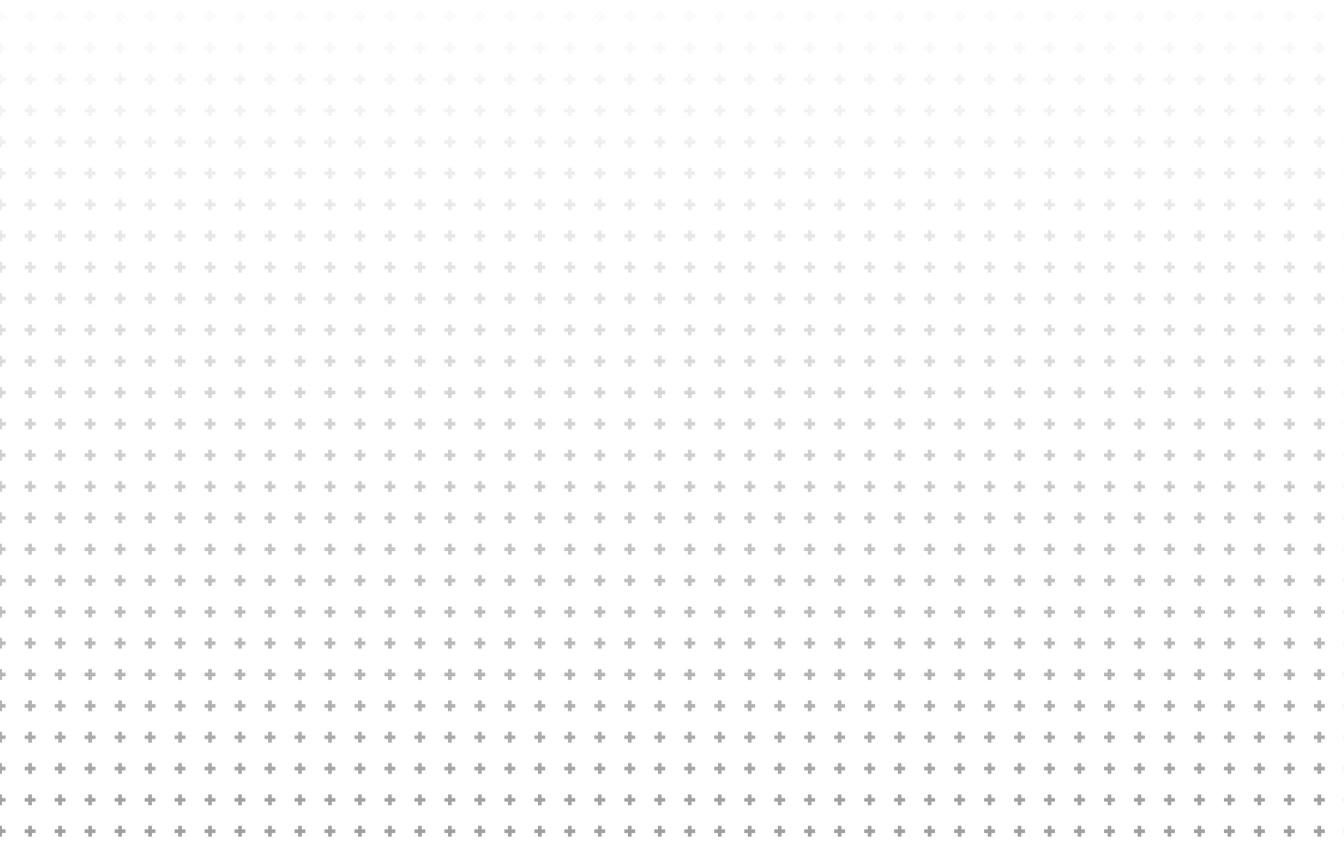
THIS IS CRAZY! I'LL DO IT!

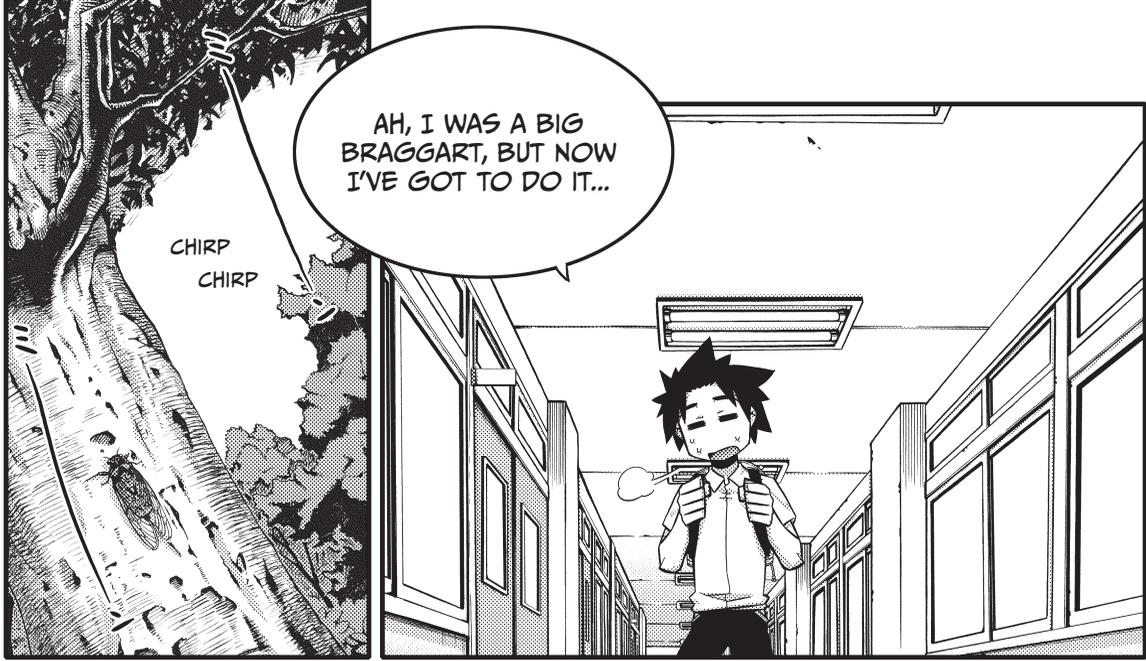
WOOF!

TEE HEE



# WHAT IS RELATIVITY?





AH, I WAS A BIG BRAGGART, BUT NOW I'VE GOT TO DO IT...

CHIRP  
CHIRP



AT LEAST I'LL BE GETTING PRIVATE LESSONS FROM THE INTRIGUING MISS URAGA!

PHYSICS ROOM

HMMM... OH, HERE IT IS.

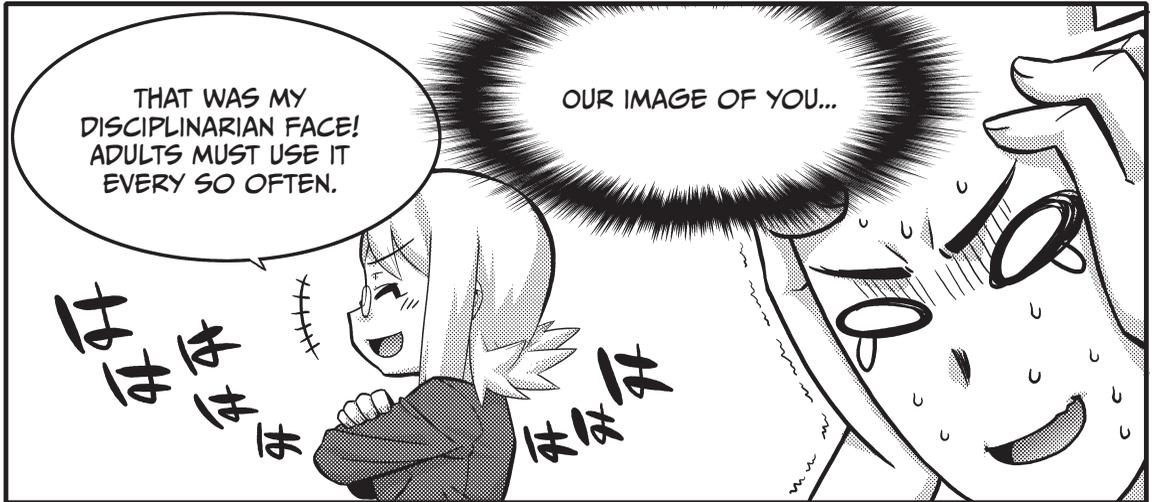


YOU'RE LATE!!!

WHAM!

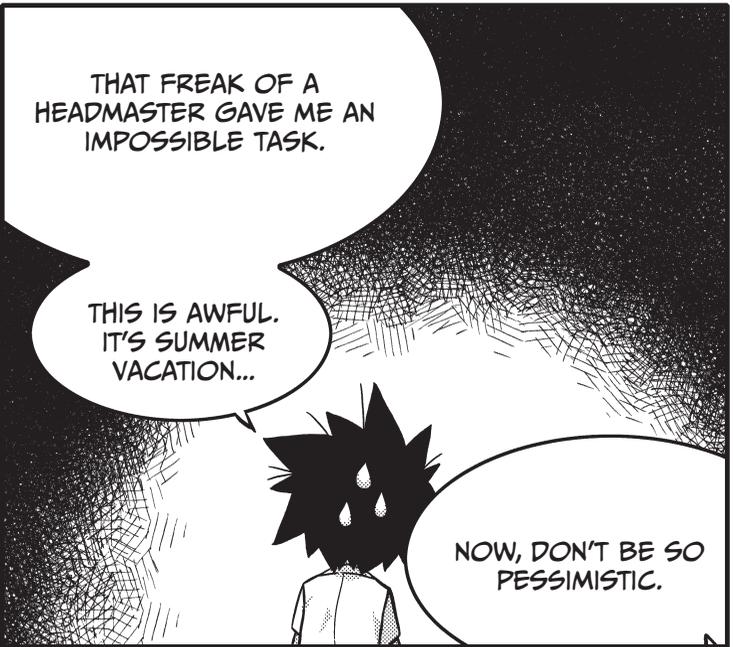
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THAT WAS MY DISCIPLINARIAN FACE! ADULTS MUST USE IT EVERY SO OFTEN.

OUR IMAGE OF YOU...



THAT FREAK OF A HEADMASTER GAVE ME AN IMPOSSIBLE TASK.

THIS IS AWFUL. IT'S SUMMER VACATION...

NOW, DON'T BE SO PESSIMISTIC.



HONESTLY, WHEN HE STARTED TALKING ABOUT RELATIVITY, I THOUGHT THAT THIS WAS MY BIG OPPORTUNITY!

WHAT DO YOU MEAN BY OPPORTUNITY?



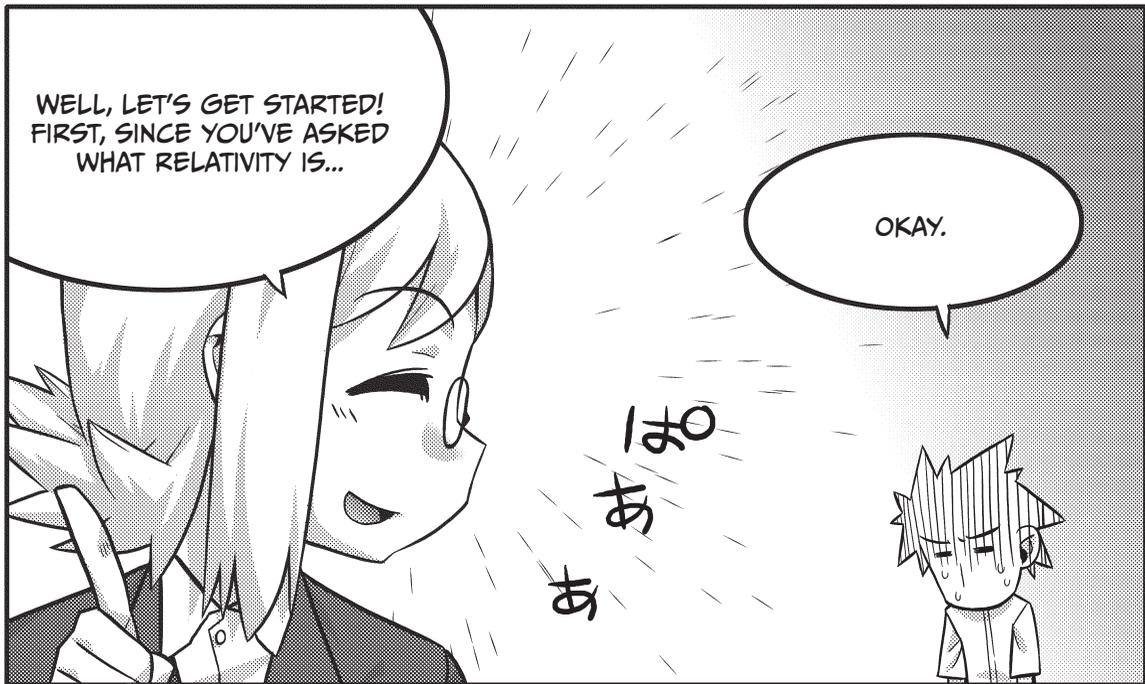
IT'S MY FIELD OF SPECIALTY. I THOUGHT I COULD USE THIS SITUATION TO RACK UP POINTS WITH THE HEADMASTER.

I'M JUST YOUR STEPPING STONE, HUH?



AND IF I GOT A HIGH EVALUATION FOR BEING "MISS URAGA WHO ENTHUSIASTICALLY PROVIDES GUIDANCE TO STUDENTS," I'D EVENTUALLY BECOME THE NEXT HEADMISTRESS.

THAT'S A BIT FAR-FETCHED, ISN'T IT?



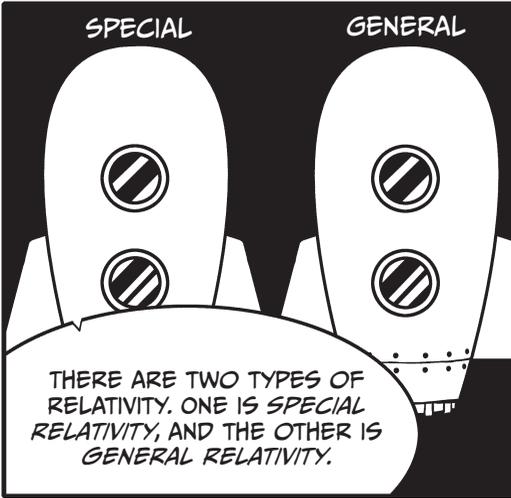
WELL, LET'S GET STARTED! FIRST, SINCE YOU'VE ASKED WHAT RELATIVITY IS...

OKAY.



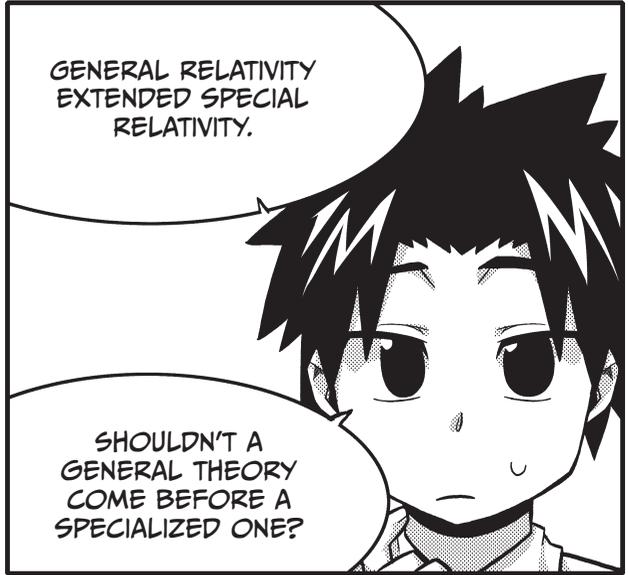
1. WHAT IS RELATIVITY?

SPECIAL GENERAL



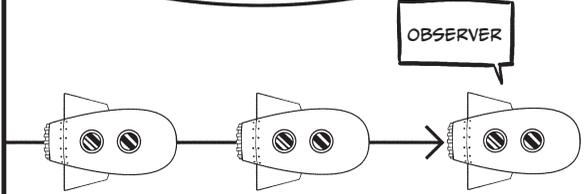
THERE ARE TWO TYPES OF RELATIVITY. ONE IS *SPECIAL RELATIVITY*, AND THE OTHER IS *GENERAL RELATIVITY*.

GENERAL RELATIVITY EXTENDED SPECIAL RELATIVITY.



SHOULDN'T A GENERAL THEORY COME BEFORE A SPECIALIZED ONE?

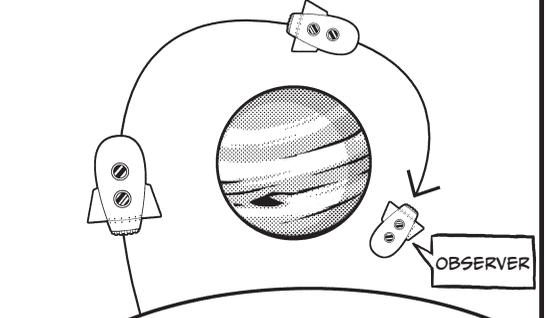
*SPECIAL RELATIVITY* IS "SPECIAL" BECAUSE IT IS A SIMPLIFICATION OF *GENERAL RELATIVITY* AND IS ONLY TRUE IN THE SPECIAL CASES WHEN THE EFFECTS OF GRAVITY AND ACCELERATION CAN BE SAFELY IGNORED.



OBSERVER

SPECIAL RELATIVITY

GENERAL RELATIVITY



OBSERVER

GENERAL RELATIVITY IS A BROADER THEORY OF RELATIVITY THAT ACCOUNTS FOR THE EFFECTS OF BOTH ACCELERATION AND GRAVITY.

IS IT SIMPLER WHEN GRAVITY OR ACCELERATION IS NOT CONSIDERED?



WELL, SURE IT IS.

THE THEORY OF RELATIVITY SAYS THAT THE PASSAGE OF TIME, DISTANCE, AND MASS DEPEND ON THE MOTION OF WHOEVER MAKES THE OBSERVATION. IN SPECIAL RELATIVITY, WE ONLY CONSIDER OBSERVERS AT REST OR WHO MOVE AT A CONSTANT VELOCITY. WE CALL THIS VANTAGE POINT AN *INERTIAL REFERENCE FRAME*.

WHEN OBSERVATIONS ARE MADE BY AN OBSERVER UNDERGOING ACCELERATION, THAT IS CALLED MAKING OBSERVATIONS FROM A *NON-INERTIAL REFERENCE FRAME*, AND WE MUST USE GENERAL RELATIVITY. LET ME EXPLAIN WHAT THESE THEORIES BROADLY MEAN.

SPECIAL RELATIVITY SAYS THAT FOR OBJECTS IN MOTION...

*TIME SLOWS DOWN, LENGTH CONTRACTS, AND MASS INCREASES.*

SLOWS DOWN, CONTRACTS?! DO THOSE THINGS REALLY OCCUR?!

CONTRACT! INCREASE!

TRANSFORMERS, GO!!

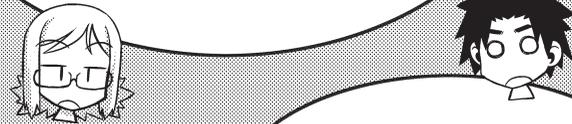
...IT'S NOT LIKE YOU ARE IMAGINING IT.

THE EFFECTS OF RELATIVITY ONLY BECOME NOTICEABLE AT SPEEDS CLOSE TO THE SPEED OF LIGHT.

THESE SPEEDS ARE EXTREMELY FAST, AND SO WE VERY RARELY OBSERVE RELATIVISTIC EFFECTS ON EARTH.

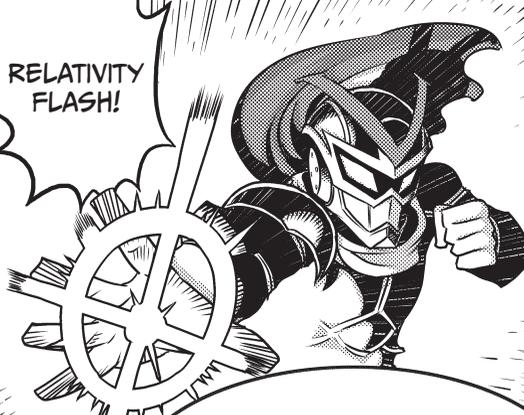
WHAT THE HECK?

GENERAL RELATIVITY SAYS THAT AN OBJECT WITH MASS CREATES GRAVITY BY AFFECTING TIME AND SPACE.



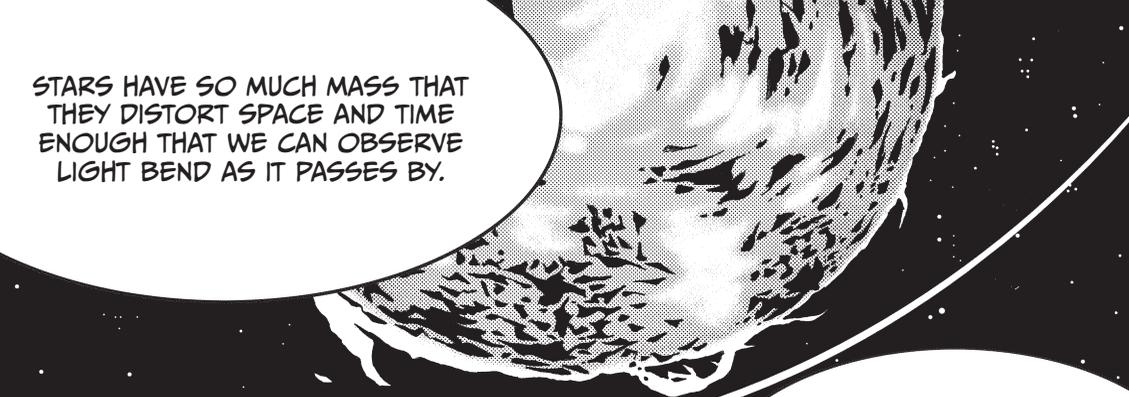
WHAT IS GRAVITY?

RELATIVITY FLASH!



I SEE. FOR EXAMPLE, LIGHT...

STARS HAVE SO MUCH MASS THAT THEY DISTORT SPACE AND TIME ENOUGH THAT WE CAN OBSERVE LIGHT BEND AS IT PASSES BY.



THE OBSERVATION OF LIGHT BENDING AS IT PASSES BY A LARGE STAR WAS ACTUALLY THE FIRST CONFIRMATION OF THE THEORY OF GENERAL RELATIVITY.

ZZZ

NOW, SINCE GENERAL RELATIVITY IS MORE ADVANCED AND DIFFICULT...



LET'S PROCEED BY FOCUSING OUR DISCUSSION ON SPECIAL RELATIVITY. YOU SHOULD APPRECIATE THIS.

OKAAAAY...



BECAUSE IF I DON'T UNDERSTAND, I'LL REALLY BE IN HOT WATER....

## 2. GALILEAN PRINCIPLE OF RELATIVITY AND NEWTONIAN MECHANICS

LET'S BEGIN WITH SOME HISTORICAL BACKGROUND SO YOU CAN UNDERSTAND RELATIVITY A LITTLE BETTER.

ㄋ ㄋ

HISTORICAL BACKGROUND?

IT WILL PROBABLY BE EASIER FOR YOU TO UNDERSTAND IF I TELL YOU HOW THE THEORY OF RELATIVITY ORIGINATED.

BY THE WAY, SINCE I'LL GLOSS OVER SOME DETAILS, I HOPE YOU'LL FORGIVE ME IF MY EXPLANATION LACKS A LITTLE SCIENTIFIC PRECISION.

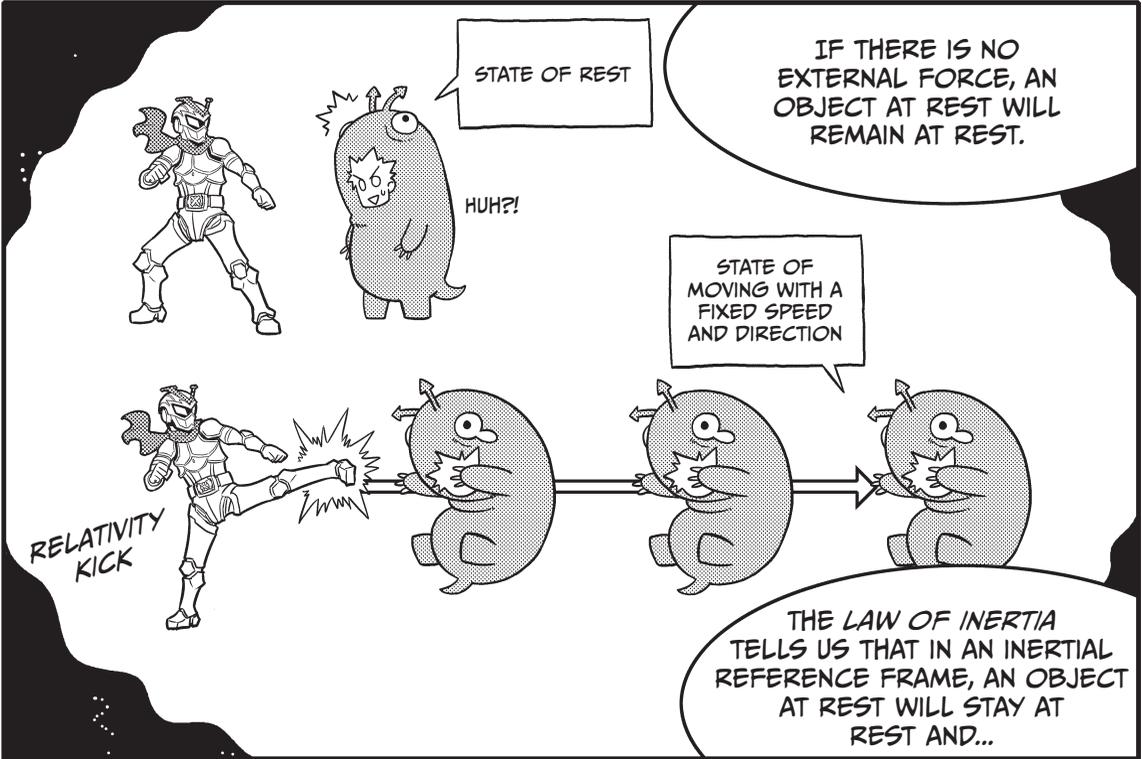
WELL, I GUESS IT'S OKAY AS LONG AS I GET THE GENERAL IDEA.

FIRST, MORE THAN 300 YEARS BEFORE EINSTEIN MADE HIS APPEARANCE...

Galileo Galilei

...THERE WAS THE GALILEAN PRINCIPLE OF RELATIVITY DISCOVERED BY GALILEO GALILEI.

RELATIVITY... PRINCIPLE?



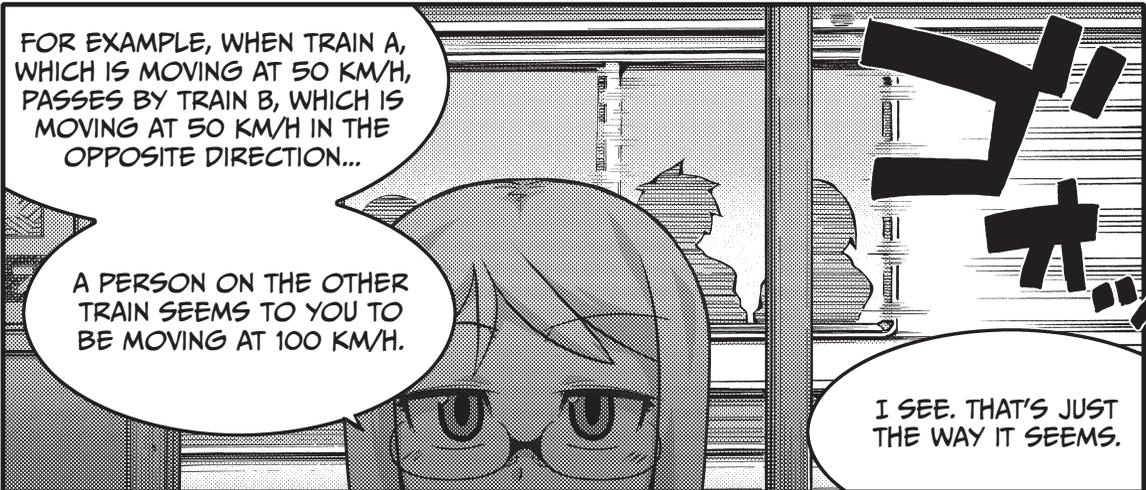
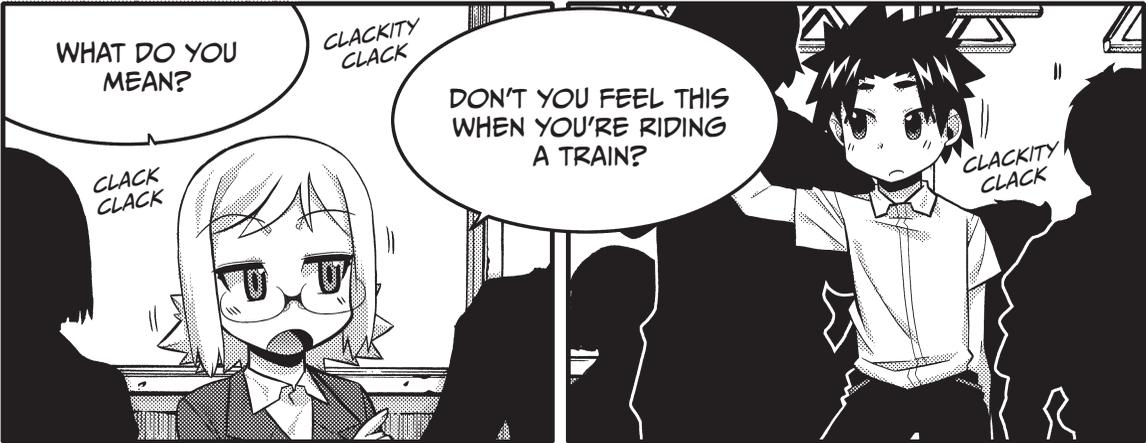
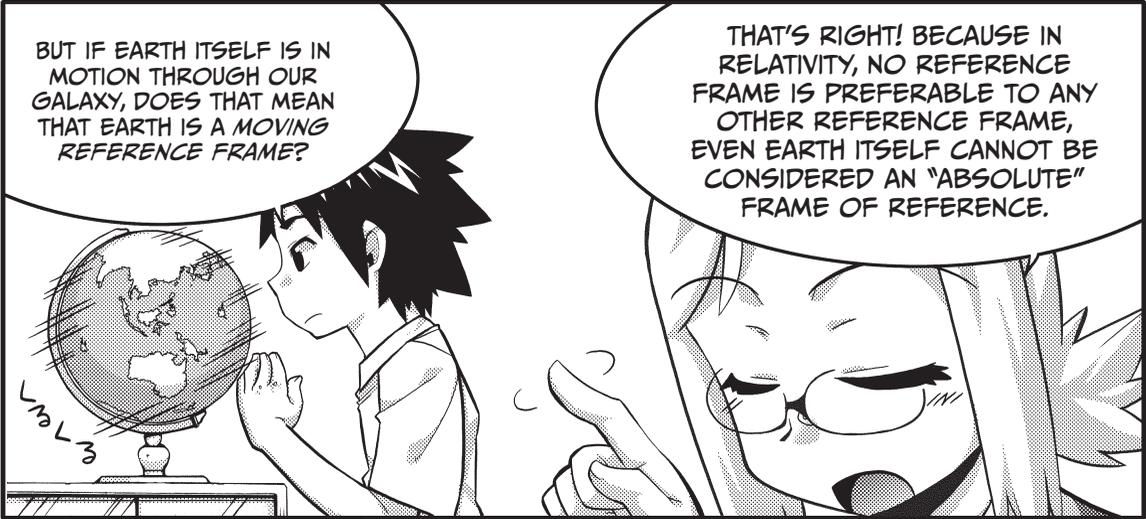
THE GALILEAN PRINCIPLE OF RELATIVITY TELLS US THAT NEWTON'S LAWS ARE THE SAME IN ANY INERTIAL REFERENCE FRAME THAT WE MIGHT CHOOSE. IN OTHER WORDS, NO MATTER WHERE AN OBSERVER IS IN THE UNIVERSE OR HOW FAST SHE IS MOVING, THE LAWS OF PHYSICS WILL NEVER CHANGE.

I SORT OF UNDERSTAND IT...

FOR EXAMPLE, IF I TOSS A BALL STRAIGHT UP IN A PLACE THAT IS AT REST, IT WILL COME BACK TO MY HAND, RIGHT?

IN THE SAME WAY, IF I TOSS A BALL STRAIGHT UP IN A TRAIN THAT IS MOVING AT A CONSTANT SPEED, IT WILL ALSO RETURN TO MY HAND.

IN OTHER WORDS, IT DOES NOT MATTER IF YOU ARE AT REST OR IF YOU ARE MOVING, THE LAWS OF PHYSICS BEHAVE EXACTLY THE SAME.



THIS PERCEIVED MOTION IS THE RESULT OF THE RELATIONSHIP BETWEEN YOU AND THE OTHER PARTY.

IT IS A WAY OF THINKING REFERRED TO AS "RELATIVE."

THE EARTH AND EVERY OTHER OBJECT IN THE UNIVERSE ARE CONTINUOUSLY MOVING RELATIVE TO EACH OTHER.

IT IS ONLY IN OUR FRAME OF REFERENCE THAT WE ARE CONSIDERED TO BE AT REST.



YOU CONSIDER THE MOVEMENT OF THE OBJECT WITH YOURSELF AS THE REFERENCE POINT, RIGHT?

MMMHMMM. YOU ASSUME THAT YOU ARE AT REST AND PERCEIVE THE MOTION RELATIVELY.

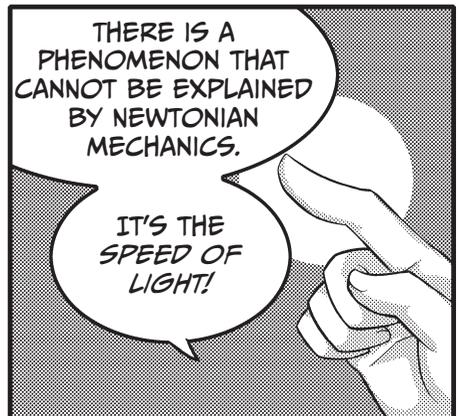
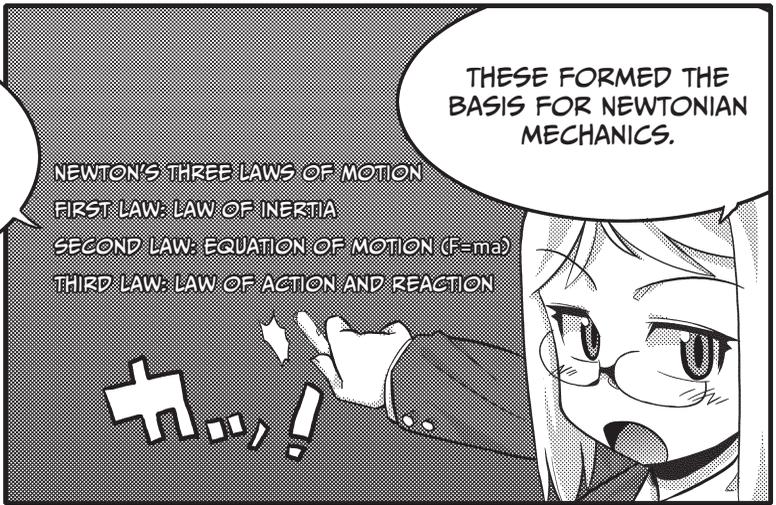
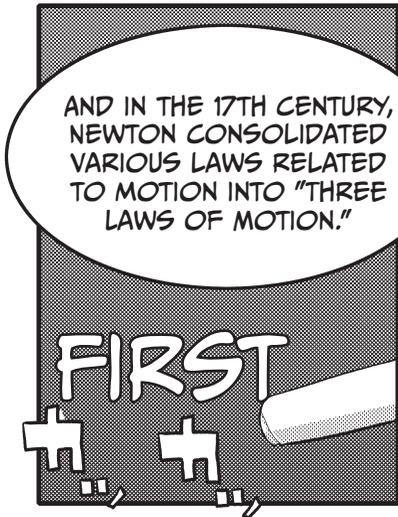
THEREFORE, PRINCIPLES OR LAWS THAT HOLD HERE ALSO APPLY ANYWHERE IN THE UNIVERSE.

YOU ARE THE REFERENCE POINT

自分が基準だ

THINKING RELATIVELY IS IMPORTANT, ISN'T IT?

CONFIDENCE!



3. MYSTERY OF THE SPEED OF LIGHT

IT CAN'T BE EXPLAINED SINCE IT'S LIKE THE RULES CREATED BY OUR CRAZY HEADMASTER...



1. HONOR THE HEADMASTER.
2. TRY TO PERFORM ONE NICE DEED A DAY FOR THE HEADMASTER.
3. DO NOT BLAME THE HEADMASTER.
4. DO NOT RECKLESSLY GIVE FOOD TO THE VICE PRINCIPAL.



”  
”  
”



HA HA HA!  
THOSE RULES WOULD  
EVEN BRING NEWTON  
TO HIS KNEES!

THOSE ARE  
SCHOOL  
REGULATIONS!

I'M TALKING  
ABOUT THE  
SPEED OF LIGHT!



OUCH!



WHAT'S THE BIG IDEA?

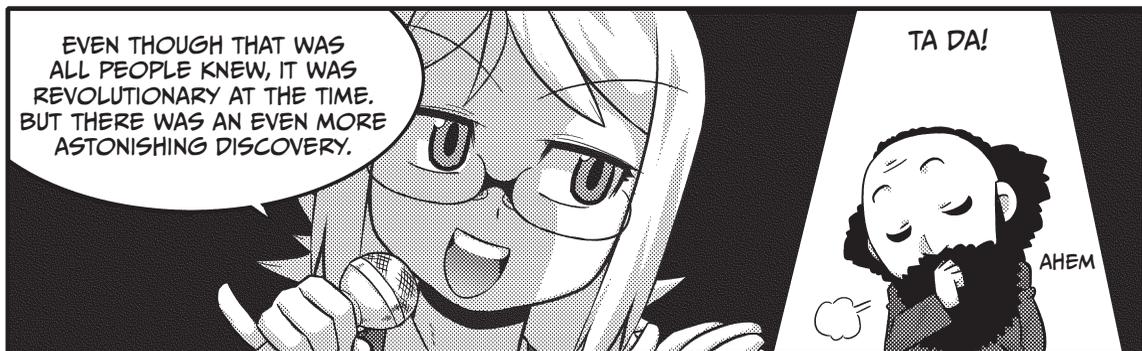
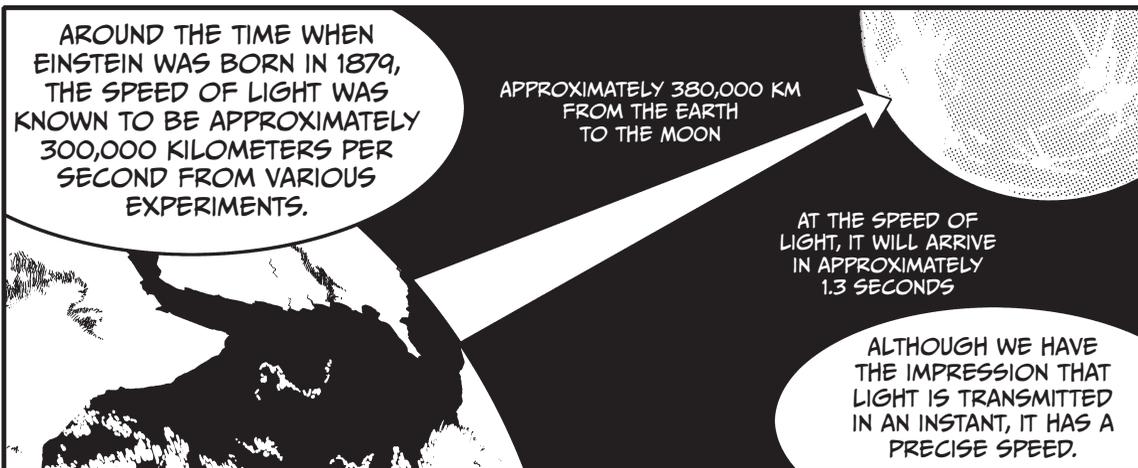
DON'T TRY TO  
CHANGE THE  
SUBJECT!

BUT ARE THE SPEED OF  
LIGHT AND RELATIVITY  
RELATED?

VERY MUCH  
SO!



IT WOULDN'T BE AN  
EXAGGERATION TO SAY  
THAT THE THEORY OF  
RELATIVITY GREW FROM  
THE MYSTERY OF THE  
SPEED OF LIGHT!



I'M GOING TO OMIT THE EQUATIONS SINCE THEY ARE DIFFICULT, BUT MAXWELL'S EQUATIONS, WHICH PERFECTLY DESCRIBED BOTH ELECTRICITY AND MAGNETISM...

INCIDENTALLY, THIS IS THE EQUATION.

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

PREDICTED THAT LIGHT WAS AN ELECTROMAGNETIC WAVE WITH A SPEED THAT WAS CONSTANT.

THESE FACTS ARE KNOWN FROM THIS EQUATION?

BECAUSE MAXWELL'S EQUATIONS PREDICTED THE SAME SPEED OF LIGHT THAT WAS MEASURED IN EXPERIMENTS, PEOPLE TOOK ITS PREDICTION THAT THE SPEED OF LIGHT WAS CONSTANT VERY SERIOUSLY.

THAT WAS A VERY IMPORTANT CONCEPT.

...VERY MUCH.

I SEE. BUT IF THE SPEED OF LIGHT IS CONSTANT, IS THERE SOME KIND OF PROBLEM?

THANK YOU...

IN NEWTONIAN MECHANICS, WHICH HAD BEEN THOUGHT TO BE ABLE TO EXPLAIN ALL LAWS OF PHYSICS, THE SPEED OF A MOVING OBJECT HAD BEEN ASSUMED TO DIFFER DEPENDING ON THE OBSERVER.

HOWEVER, HERE'S WHERE THE PROBLEM ARISES. IF THE SPEED OF LIGHT IS CONSTANT, THEN WHAT IS IT CONSTANT RELATIVE TO?

**NEWTONIAN MECHANICS**

ROCKET FLYING AT 10 KM/S

MISILE THAT WAS FIRED AT 10 KM/S

OBSERVED FROM THE ROCKET FLYING AT 10 KM/S, THE MISSILE IS GOING 10 KM/S

WHEN OBSERVED BY A PERSON AT REST, THE 10 KM/S OF THE ROCKET IS ADDED SO THAT THE MISSILE IS GOING 20 KM/S

**FOR THE SPEED OF LIGHT**

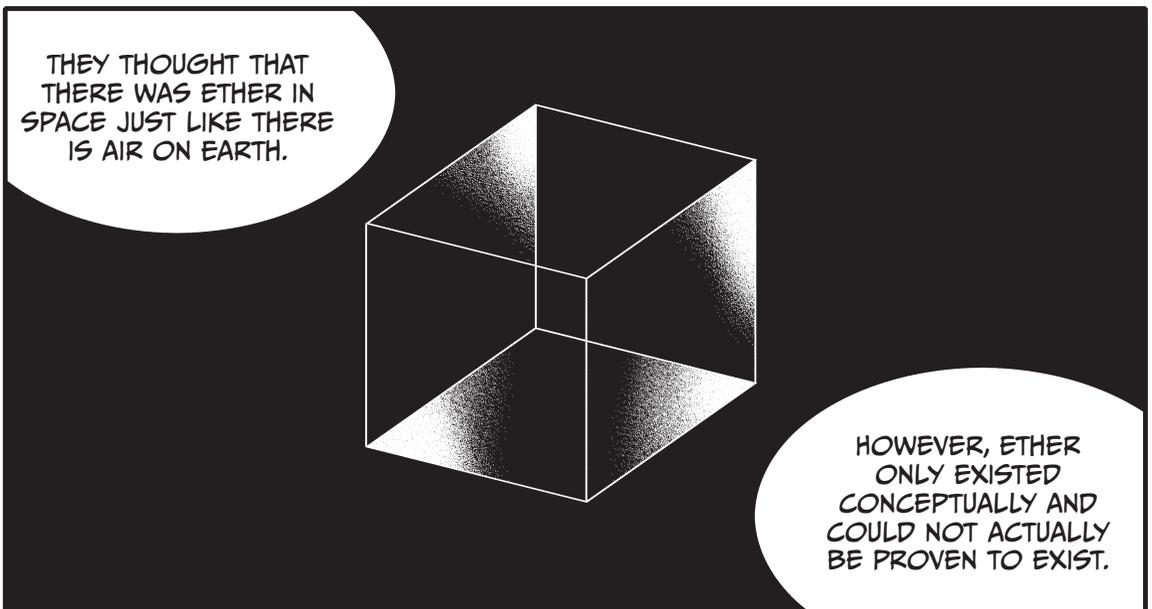
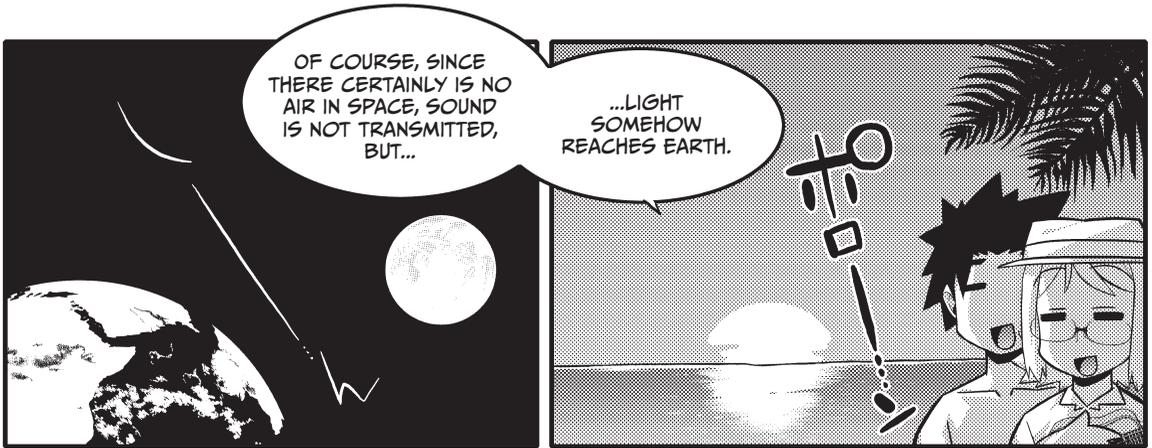
ROCKET FLYING AT 90% OF THE SPEED OF LIGHT

LIGHT EMITTED FROM THE ROCKET

OBSERVED FROM A ROCKET FLYING AT 90% OF THE SPEED OF LIGHT, LIGHT IS MOVING AT 300,000 KM/S?!

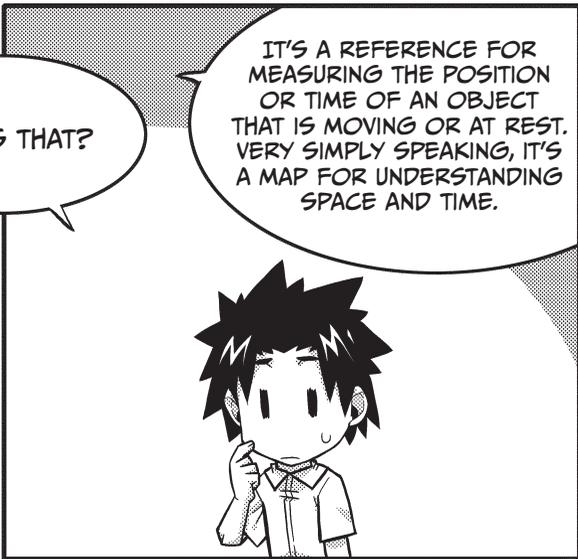
EVEN WHEN OBSERVED BY A PERSON AT REST, LIGHT IS MOVING AT 300,000 KM/S?!







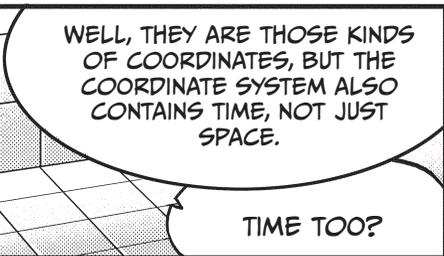
NEXT, I WILL TALK ABOUT ABSOLUTE STATIONARY SPACE, BUT FIRST, I WANT TO EXPLAIN A COORDINATE SYSTEM.



IT'S A REFERENCE FOR MEASURING THE POSITION OR TIME OF AN OBJECT THAT IS MOVING OR AT REST. VERY SIMPLY SPEAKING, IT'S A MAP FOR UNDERSTANDING SPACE AND TIME.

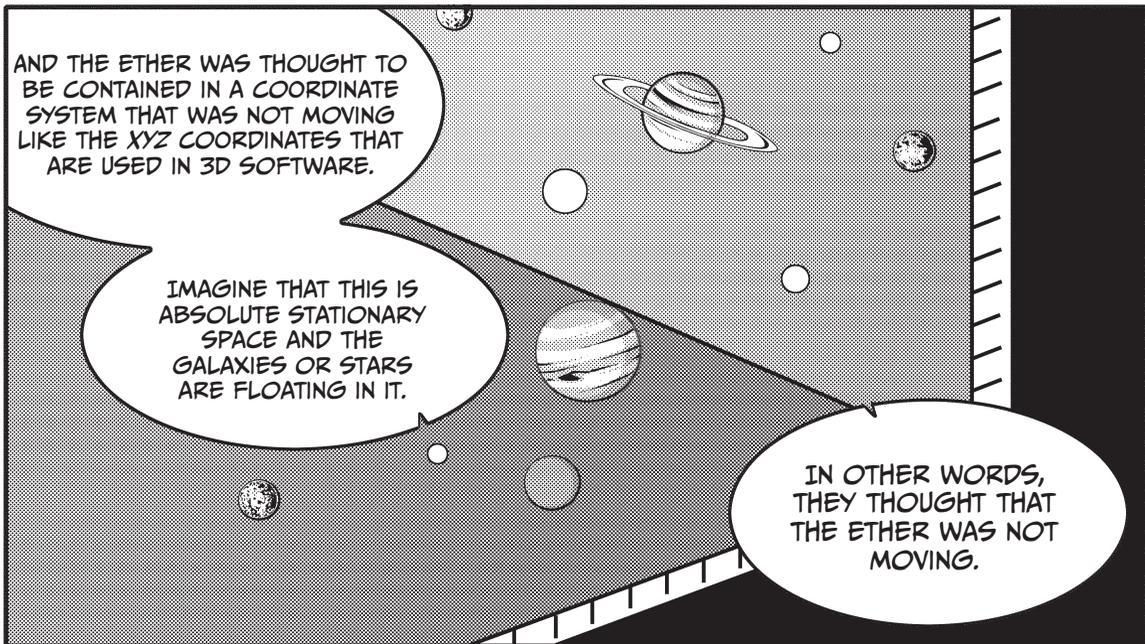


ARE THEY COORDINATES LIKE THE X- OR Y-AXIS COORDINATES?



WELL, THEY ARE THOSE KINDS OF COORDINATES, BUT THE COORDINATE SYSTEM ALSO CONTAINS TIME, NOT JUST SPACE.

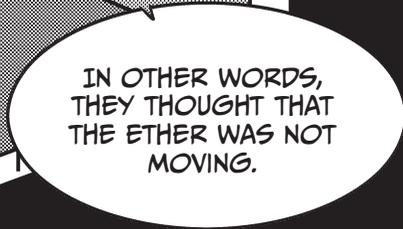
TIME TOO?



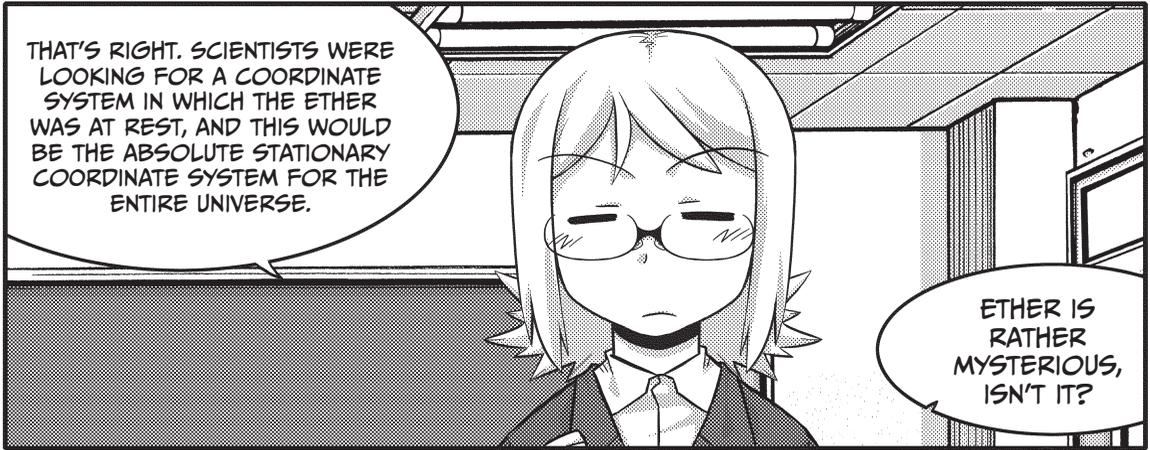
AND THE ETHER WAS THOUGHT TO BE CONTAINED IN A COORDINATE SYSTEM THAT WAS NOT MOVING LIKE THE XYZ COORDINATES THAT ARE USED IN 3D SOFTWARE.



IMAGINE THAT THIS IS ABSOLUTE STATIONARY SPACE AND THE GALAXIES OR STARS ARE FLOATING IN IT.

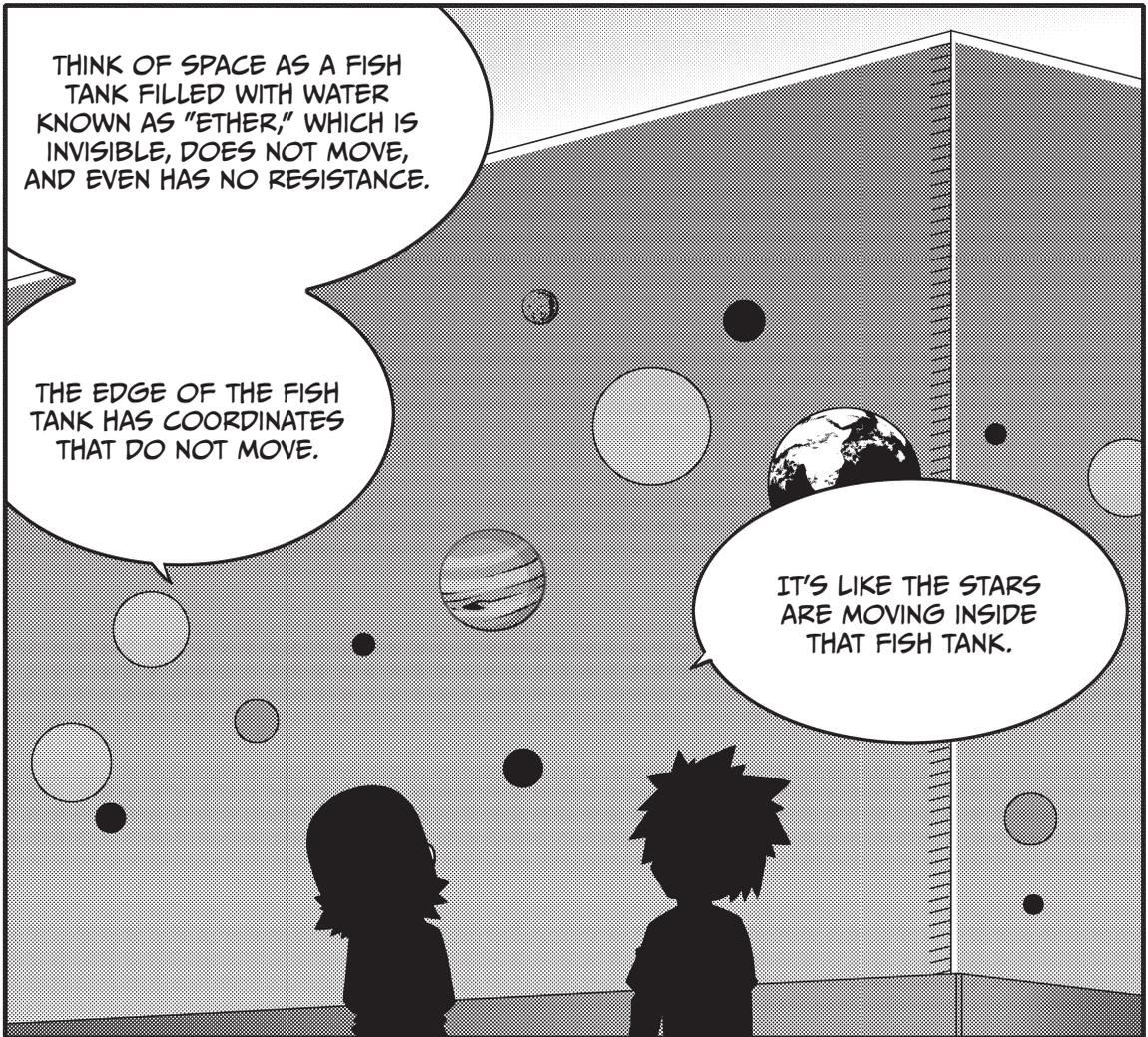


IN OTHER WORDS, THEY THOUGHT THAT THE ETHER WAS NOT MOVING.



THAT'S RIGHT. SCIENTISTS WERE LOOKING FOR A COORDINATE SYSTEM IN WHICH THE ETHER WAS AT REST, AND THIS WOULD BE THE ABSOLUTE STATIONARY COORDINATE SYSTEM FOR THE ENTIRE UNIVERSE.

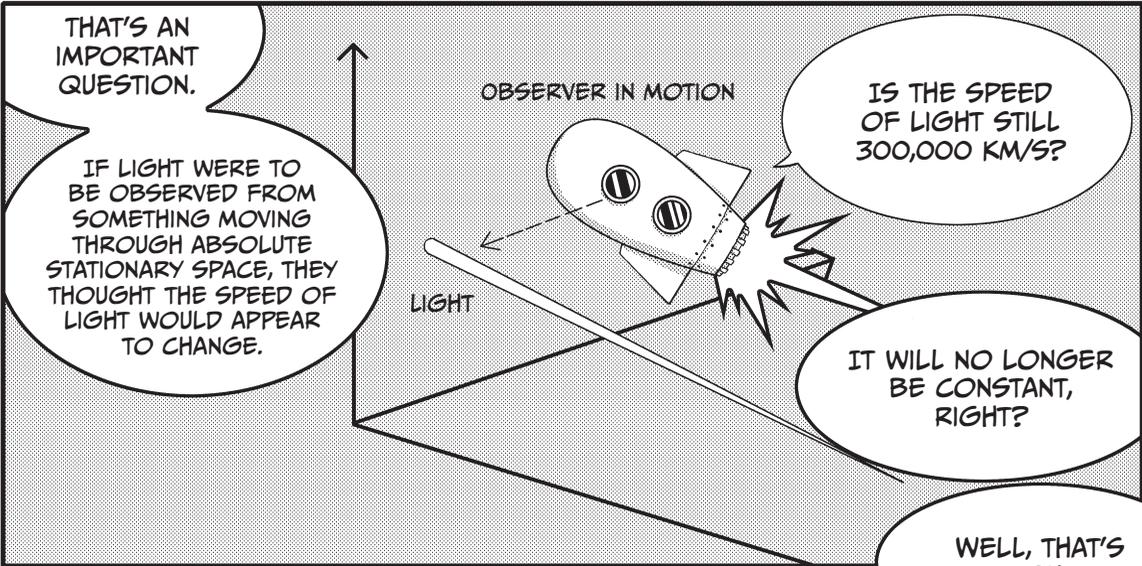
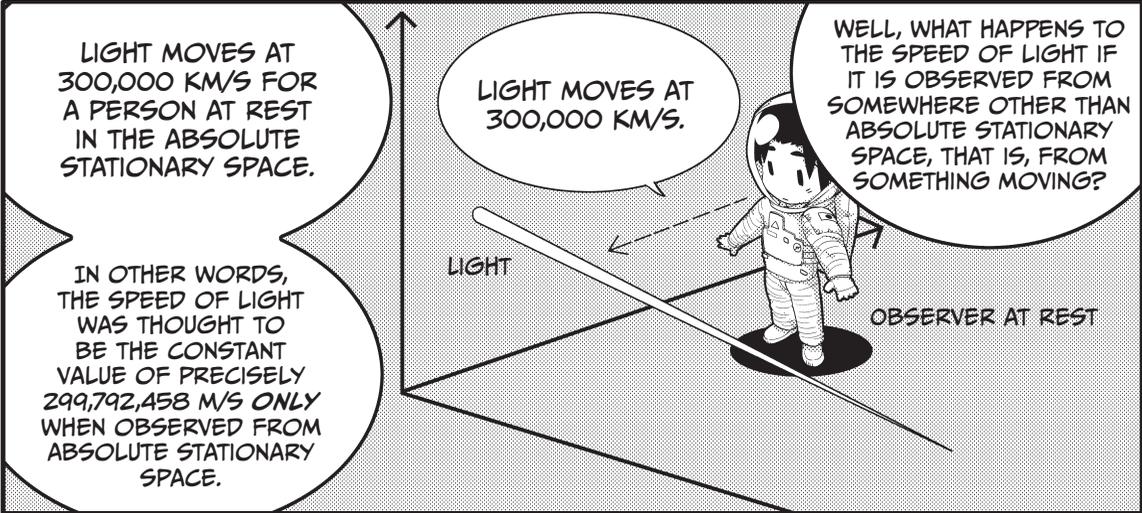
ETHER IS RATHER MYSTERIOUS, ISN'T IT?

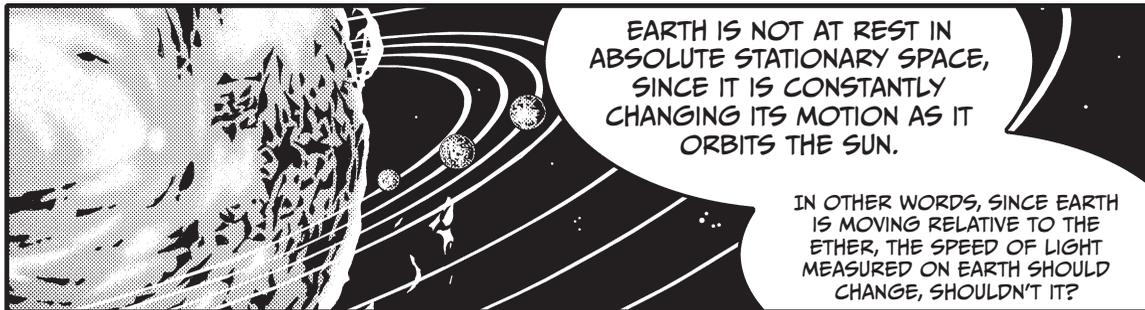


THINK OF SPACE AS A FISH TANK FILLED WITH WATER KNOWN AS "ETHER," WHICH IS INVISIBLE, DOES NOT MOVE, AND EVEN HAS NO RESISTANCE.

THE EDGE OF THE FISH TANK HAS COORDINATES THAT DO NOT MOVE.

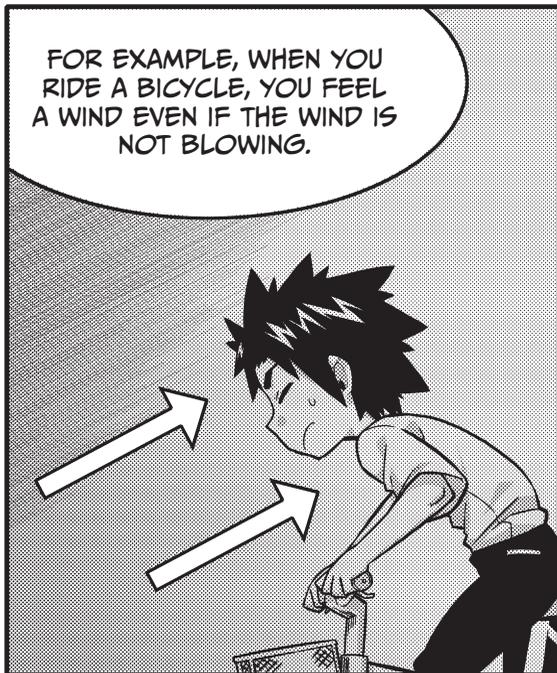
IT'S LIKE THE STARS ARE MOVING INSIDE THAT FISH TANK.



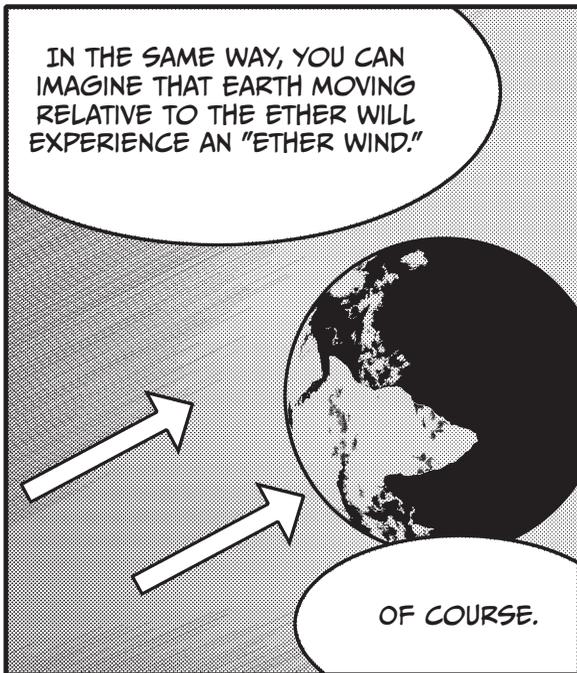


EARTH IS NOT AT REST IN ABSOLUTE STATIONARY SPACE, SINCE IT IS CONSTANTLY CHANGING ITS MOTION AS IT ORBITS THE SUN.

IN OTHER WORDS, SINCE EARTH IS MOVING RELATIVE TO THE ETHER, THE SPEED OF LIGHT MEASURED ON EARTH SHOULD CHANGE, SHOULDN'T IT?



FOR EXAMPLE, WHEN YOU RIDE A BICYCLE, YOU FEEL A WIND EVEN IF THE WIND IS NOT BLOWING.



IN THE SAME WAY, YOU CAN IMAGINE THAT EARTH MOVING RELATIVE TO THE ETHER WILL EXPERIENCE AN "ETHER WIND."

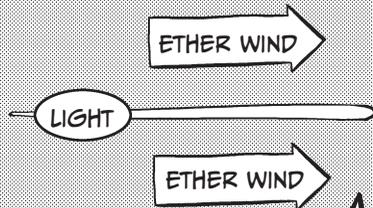
OF COURSE.



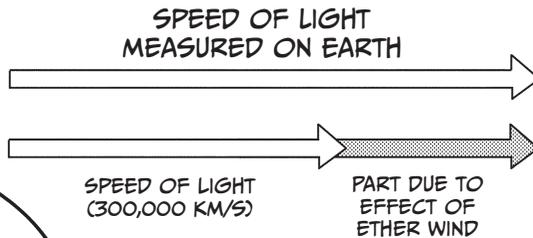
THEREFORE, IF EARTH IS SUBJECT TO AN ETHER WIND...

SHOULDN'T THE SPEED OF LIGHT ON EARTH DEVIATE FROM 300,000 KM/S BECAUSE OF ITS EFFECT?

ER, DOES THAT MEAN THAT IF THERE WERE A "HEADWIND" RELATIVE TO THE ETHER, FOR EXAMPLE...



...THE LIGHT OBSERVED FROM EARTH WOULD BE "300,000 KM/S + AN ETHER WIND PUSH" ACCORDING TO NEWTONIAN MECHANICS?



THAT'S RIGHT.

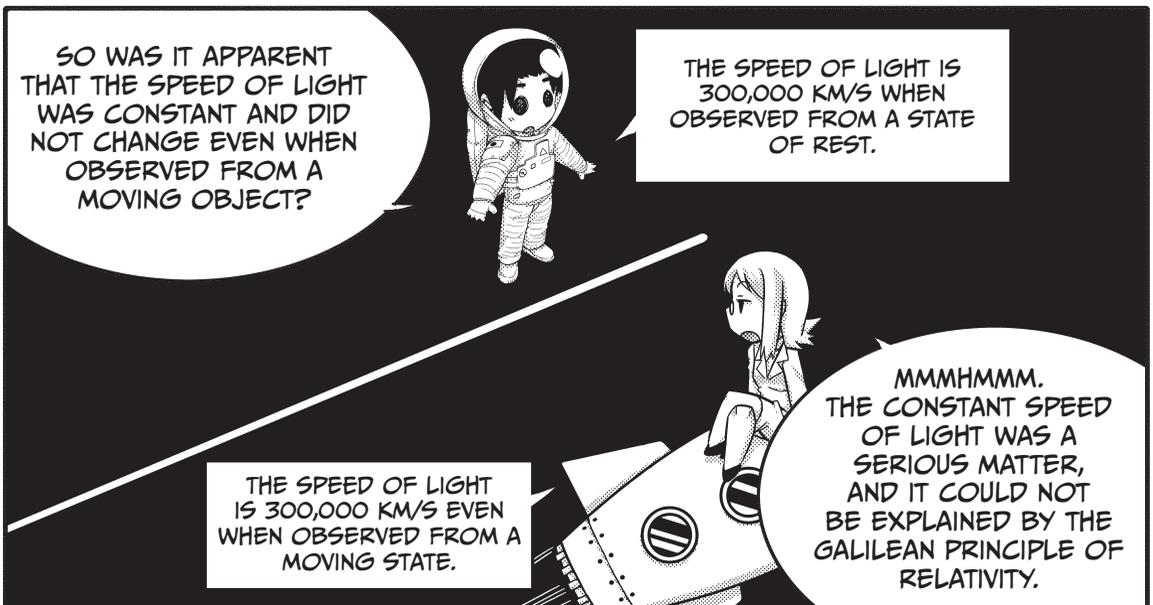
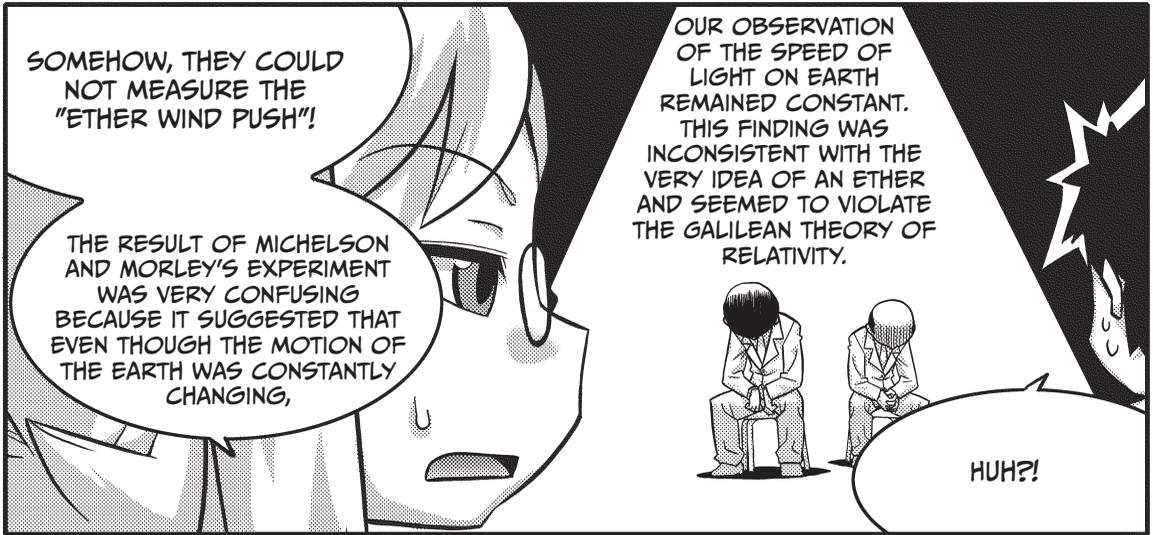
TWO PHYSICISTS NAMED MICHELSON AND MORLEY ACTUALLY TRIED TO MEASURE THE "ETHER WIND PUSH" BY USING A DEVICE FOR DETECTING THE PRECISE SPEED OF LIGHT.

Albert  
Abraham  
Michelson



Edward  
Williams  
Morley

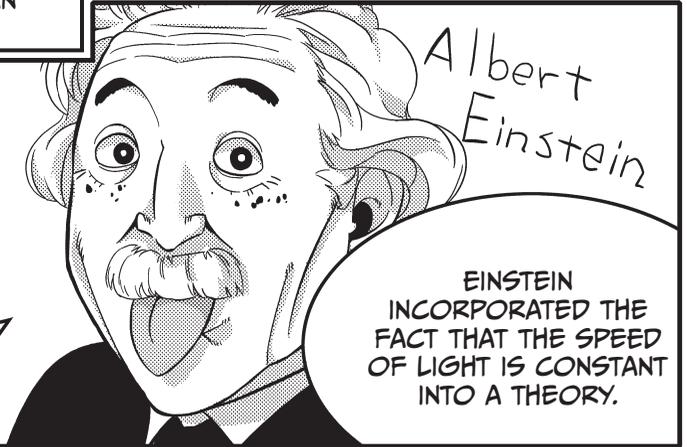
THAT WAS A MAGNIFICENT EXPERIMENT, WASN'T IT?



4. EINSTEIN DISCARDED NEWTONIAN MECHANICS

THEN, THE FAMOUS EINSTEIN ARRIVED ON THE SCENE!

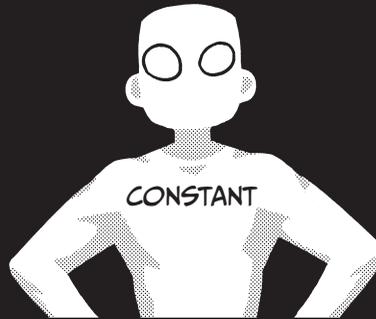
OH!  
FINALLY!



EINSTEIN INCORPORATED THE FACT THAT THE SPEED OF LIGHT IS CONSTANT INTO A THEORY.

IN OTHER WORDS, HE DISCARDED THE NEWTONIAN MECHANICAL CONCEPTS BASED ON THE GALILEAN PRINCIPLE OF RELATIVITY...

AND POSTULATED THAT THE SPEED OF LIGHT WAS CONSTANT REGARDLESS OF WHO WAS VIEWING IT.

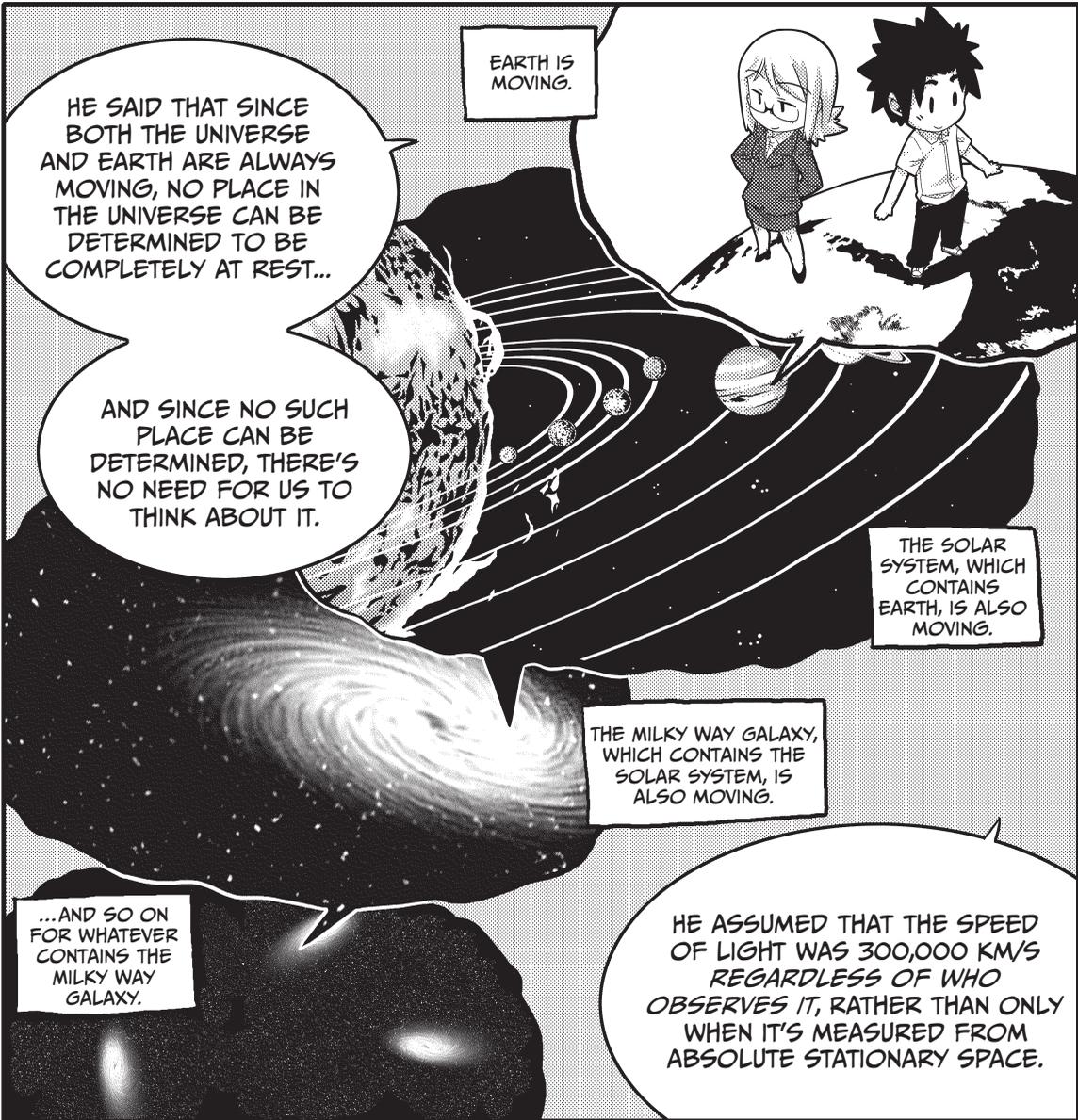


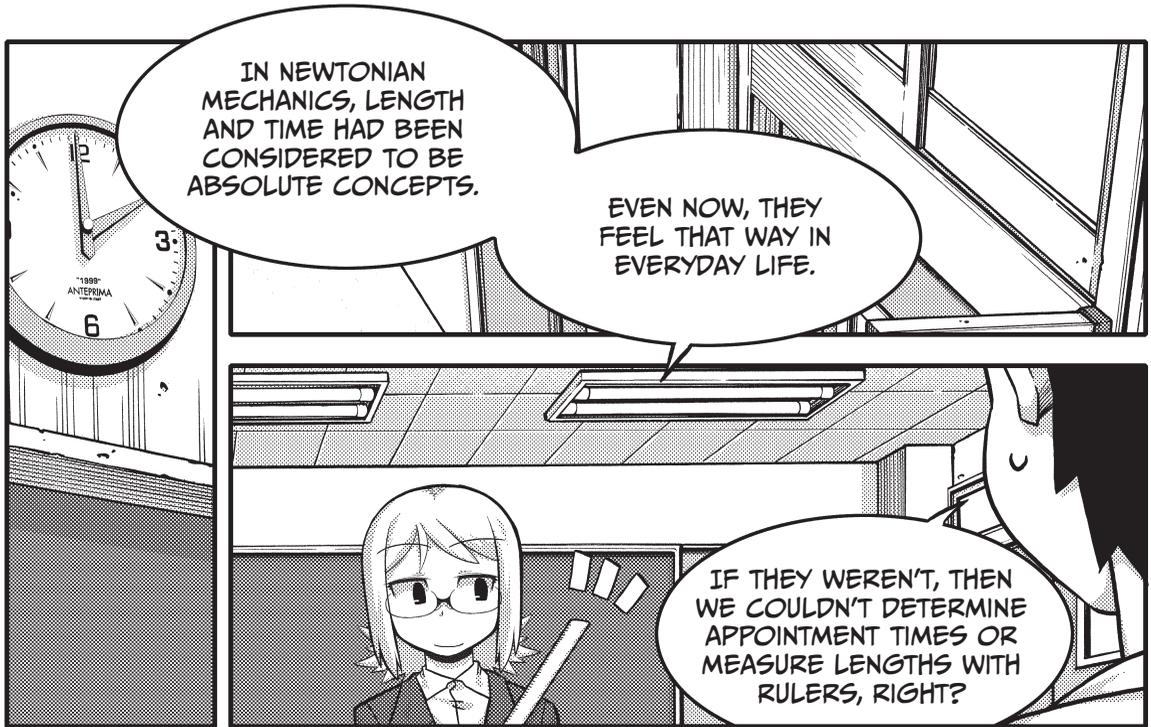
THAT'S A UNIQUE WAY OF LOOKING AT IT!

IN ADDITION, HE PROPOSED A NEW PRINCIPLE OF RELATIVITY TO BE SUBSTITUTED FOR THE GALILEAN PRINCIPLE OF RELATIVITY. THIS NEW PRINCIPLE OF RELATIVITY SAID THAT ALL PHYSICAL LAWS, INCLUDING THOSE RELATED TO LIGHT, HOLD IN EXACTLY THE SAME WAY REGARDLESS OF THE INERTIAL FRAME.

THIS IS EINSTEIN'S SPECIAL THEORY OF RELATIVITY!







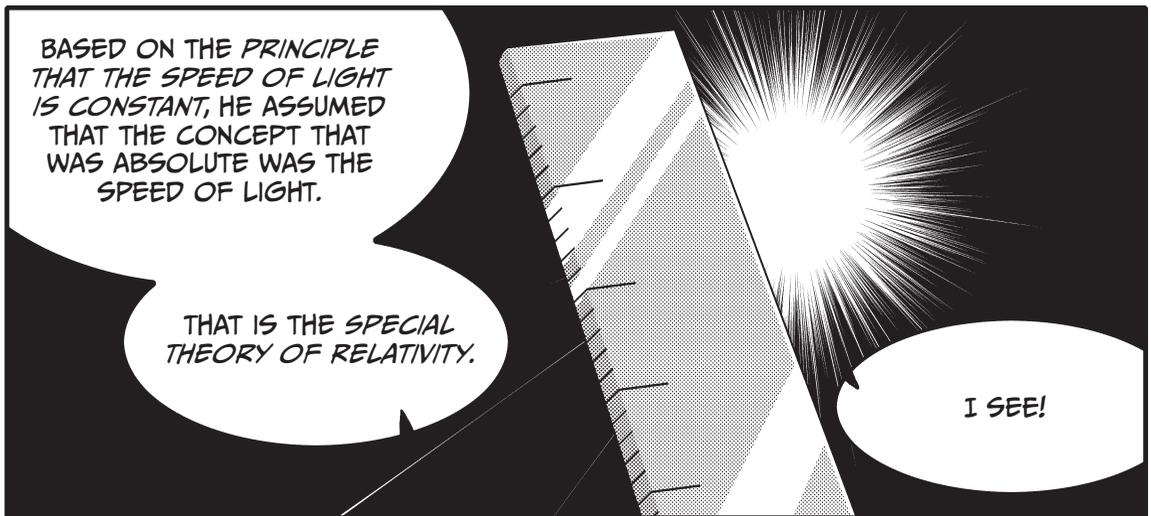
IN NEWTONIAN MECHANICS, LENGTH AND TIME HAD BEEN CONSIDERED TO BE ABSOLUTE CONCEPTS.

EVEN NOW, THEY FEEL THAT WAY IN EVERYDAY LIFE.

IF THEY WEREN'T, THEN WE COULDN'T DETERMINE APPOINTMENT TIMES OR MEASURE LENGTHS WITH RULERS, RIGHT?



NEVERTHELESS, EINSTEIN OVERTURNED THOSE CONCEPTS!



BASED ON THE PRINCIPLE THAT THE SPEED OF LIGHT IS CONSTANT, HE ASSUMED THAT THE CONCEPT THAT WAS ABSOLUTE WAS THE SPEED OF LIGHT.

THAT IS THE SPECIAL THEORY OF RELATIVITY.

I SEE!

SPEED IS DISTANCE  
TRAVELED ÷ TIME, RIGHT?

THEREFORE, SINCE THE SPEED  
OF LIGHT IS CONSTANT IN ANY  
REFERENCE FRAME, DISTANCE AND  
TIME VARY DEPENDING ON THE  
MOTION OF THE OBSERVER.  
THIS IS A MAJOR PREMISE OF  
SPECIAL RELATIVITY!

EVEN THOUGH YOU SAY  
THIS, IT FEELS STRANGE,  
BUT THIS REALLY  
HAPPENS, RIGHT?

"TIME" AND "SPACE,"  
WHICH HAD BEEN  
THOUGHT TO BE  
SEPARATE THINGS IN  
NEWTONIAN MECHANICS...

...WERE NOW CONSIDERED  
TOGETHER IN THE FORM  
OF A NEW, AMAZING  
COORDINATE SYSTEM  
CALLED SPACE-TIME.

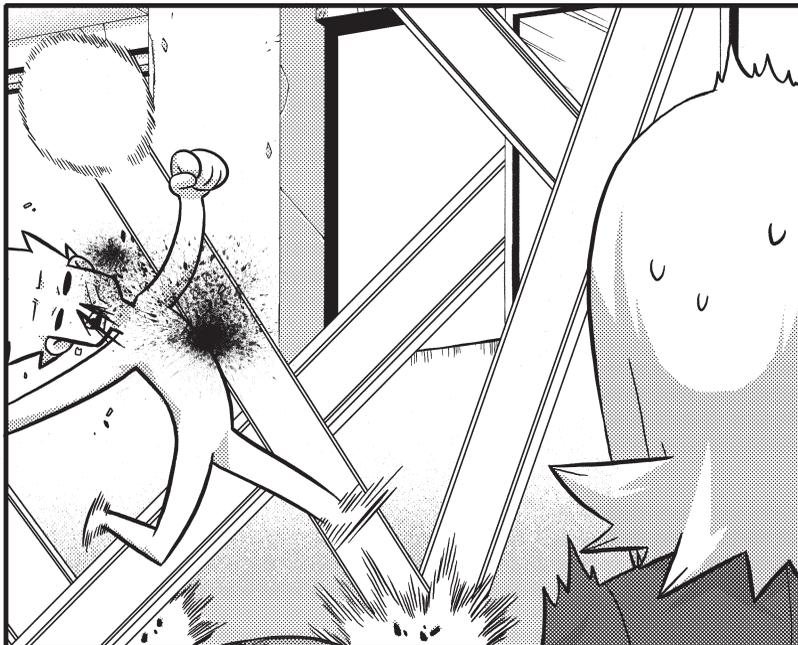
WHEN YOU SAY THAT,  
IT SOUNDS KIND OF  
AMAZING.

IT REALLY IS AMAZING!  
NOW I'LL TELL YOU WHAT  
SPECIAL RELATIVITY IS  
ALL ABOUT.

OKAY!  
PLEASE DO!

I'M SUDDENLY  
OVERFLOWING WITH  
AMBITION!

HE'S GETTING  
A LITTLE TOO  
EXCITED...



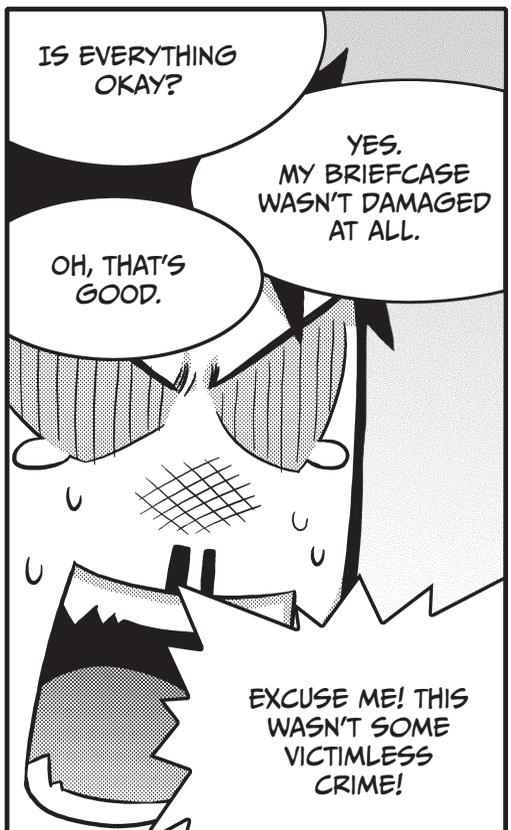
WHAT'S THIS?  
A RHYTHMIC  
GYMNASTICS BALL?!

ARGH, IT'S  
BOUNCING AROUND!  
MY BRIEFCASE IS IN  
DANGER!

HEY, SHOULDN'T YOU  
WORRY ABOUT YOUR  
STUDENT FIRST?



EXCUSE ME,  
MY BALL...

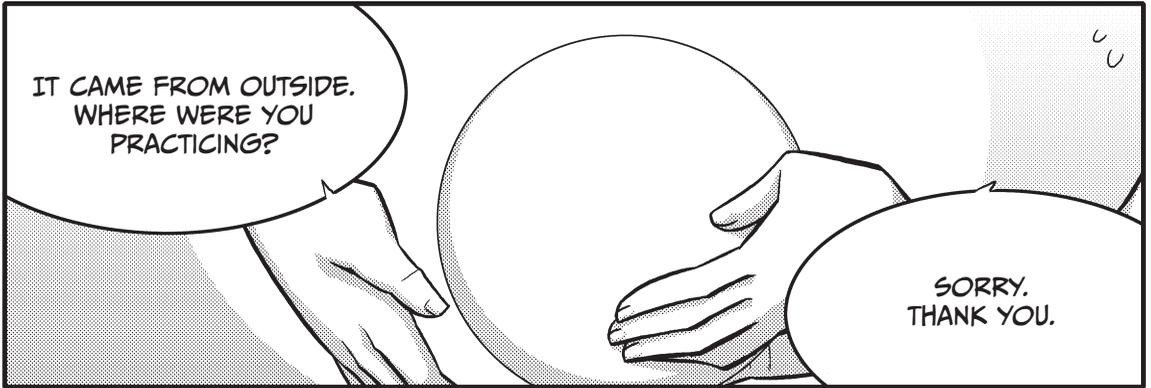


IS EVERYTHING  
OKAY?

YES.  
MY BRIEFCASE  
WASN'T DAMAGED  
AT ALL.

OH, THAT'S  
GOOD.

EXCUSE ME! THIS  
WASN'T SOME  
VICTIMLESS  
CRIME!



# WHAT IS LIGHT?

Maxwell's equations tell us that light is an electromagnetic wave. The color of light is determined by the wavelength of the electromagnetic wave. Red light has a wavelength of 630 nm, and blue light has a shorter wavelength of approximately 400 nm, where one nanometer (1 nm) = one billionth of a meter ( $10^{-9}$  m). Electromagnetic radiation at different wavelengths takes many forms, such as radio waves, X-rays, and gamma ( $\gamma$ ) rays (see Figure 1-1).

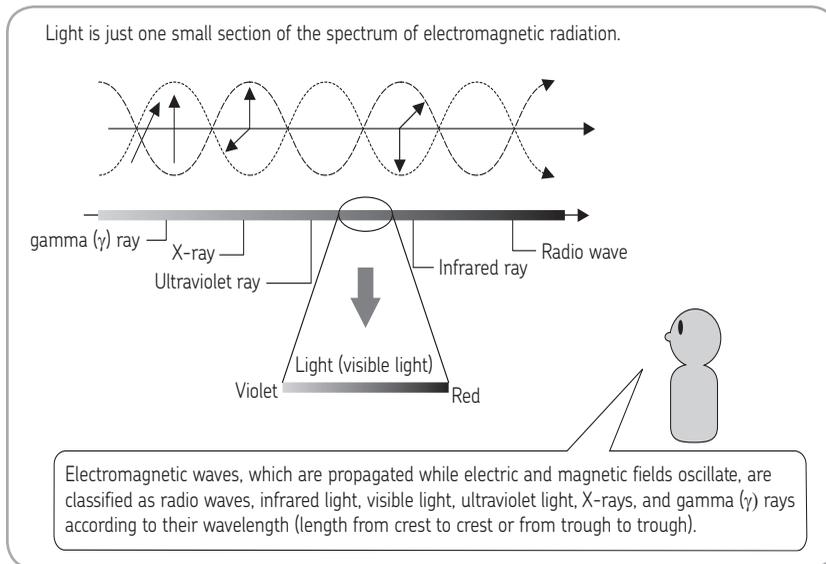


Figure 1-1: Light is an electromagnetic wave.

Although light may seem common enough—it is all around us, after all—it is fundamental to both relativity and quantum theory, the cornerstones of modern physics.

But before we delve into light's true nature, let's introduce the properties of light that have been known for a long time.

First, you know that light is *reflected* by a mirror or the surface of water. You also know about the *refraction* of light—you only need to look at your feet the next time you take a bath or see how your straw “bends” when you put it in a glass of water. Any change in medium changes a wave's direction, due to a change in the wave's speed through that medium.

Some mediums refract light of different wavelengths different amounts. In other words, light of different colors is bent to different degrees, a property known as *dispersion*. This causes white light, which consists of light of all colors, to be spread out into a spectrum of light from red to violet. We can see the seven colors of a rainbow because of dispersion.

These properties of reflection, refraction, and dispersion have been used to create precision camera lenses and telescopes. Figure 1-2 shows what happens to light when it is reflected, refracted, or dispersed.

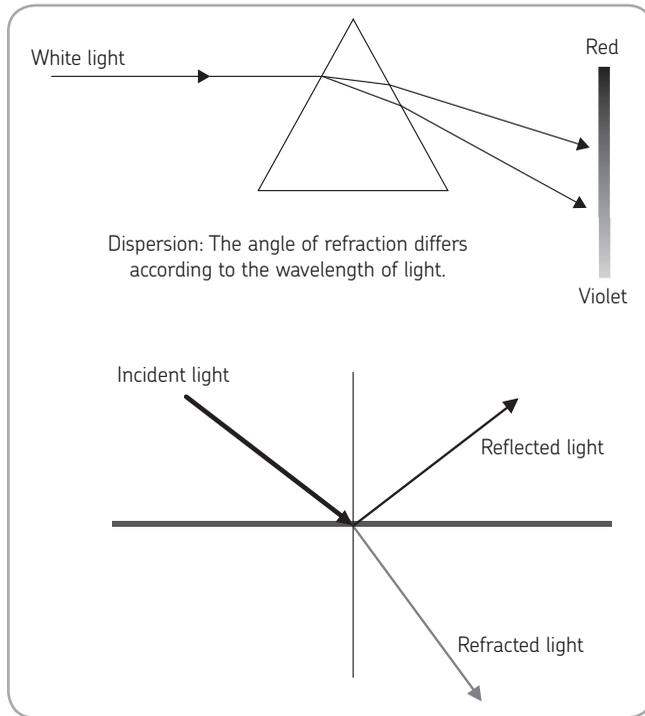


Figure 1-2: Dispersion, reflection, and refraction

Next, more subtle phenomena called *interference* and *diffraction* can be observed. These phenomena stem from the fact that light is a wave. Interference describes what happens when two light waves come together. When the two waves come together, the result is either *constructive interference*, where the waves' amplitudes are added together, or *destructive interference*, where one wave's amplitude is subtracted from the other's. Figure 1-3 shows the different kinds of interference.

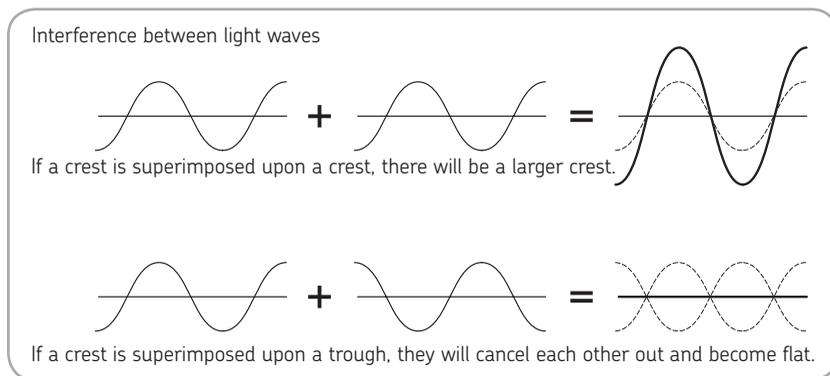


Figure 1-3: Interference can make waves stronger or weaker.

Diffraction can be observed when light passes through a tiny hole about the same size as the wavelength of the light. Due to the constructive and destructive interference of different parts of the light wave with itself, passing through a tiny aperture can cause the light to spread out or bend, as shown in Figure 1-4. Diffraction is often what limits the resolution of microscopes.

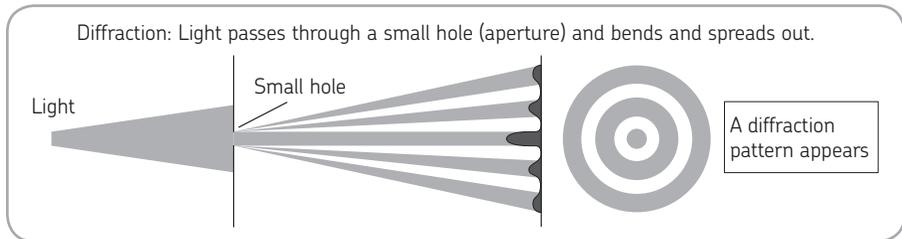


Figure 1-4: Diffraction comes about from interference.

Another property of light is called *polarization*, a property that describes the orientation of the transverse electric and magnetic components of the electromagnetic wave. This property is very useful; it allows special filters to be made (called *polarizing filters*) that allow only light with a specific polarization to pass (see Figure 1-5).

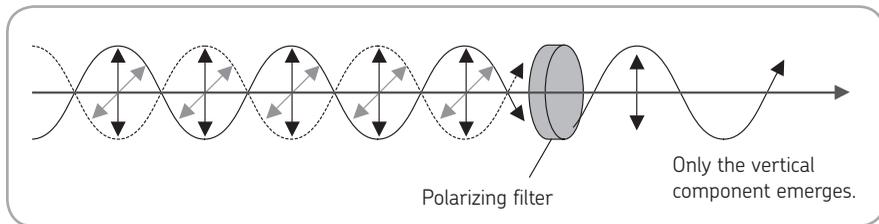


Figure 1-5: Polarization

In *scattering*, light collides with dust and other particles in the air, thereby changing direction (see Figure 1-6). Since blue light (with shorter wavelengths) is scattered by water molecules in the air more than red light (with longer wavelengths), the sky appears blue.

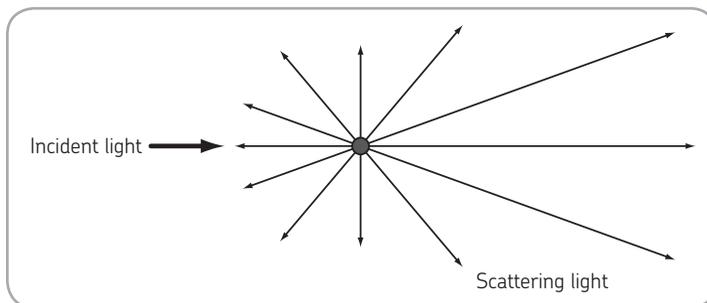


Figure 1-6: Scattering

## LIGHT IS CONSTANT (AND THEY PROVE IT EVERY DAY IN A LAB CALLED SPRING-8)

Various tests have been conducted to verify that the speed of light is truly constant. This is important because it is one of the fundamental premises of relativity.

One way that we can test this property is to measure the speed of light coming from an object that is moving very fast. If the speed of light is not constant, the Newtonian notion of “adding” relative velocities predicts that light coming from an object moving towards the observer will be the speed of light plus the speed of the moving object; for example, if the object is moving near the speed of light, then the light from the object should be moving nearly twice the speed of light. If the speed of light is constant, on the other hand, then the light coming from the fast-moving object will just be the speed of light. Measurements confirm that the speed of light is always the same, regardless of the speed of the object from which it comes (see Figure 1-7).

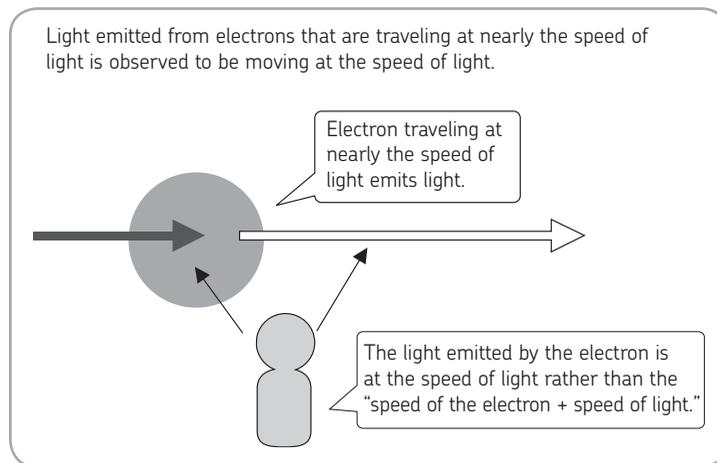


Figure 1-7: Verification that the speed of light is constant at SPring-8

Moving objects near the speed of light for these experiments is extremely difficult, and these experiments are performed at very specialized scientific facilities. SPring-8 is a synchrotron radiation facility in Japan's Hyogo Prefecture that performs experiments by smashing together electrons traveling at extremely fast speeds (99.9999998 percent of the speed of light). Besides verifying that the speed of light is constant, these experiments help scientists uncover the basic building blocks of matter.

## WHAT'S SIMULTANEOUS DEPENDS ON WHOM YOU ASK! (SIMULTANEITY MISMATCH)

If we consider the principle that “the speed of light is constant,” various phenomena appear strange. One of these is the phenomenon called the *simultaneity mismatch*, which means that what is simultaneous for me is not the same as what is simultaneous for you.

I can imagine that you are thinking, “What in the world are you saying?” So let’s consider the concept of “simultaneous” again. We will compare the case of Newtonian velocity addition (nonrelativistic addition of velocity) with the case in which the speed of light is constant (relativistic addition of velocity).

Consider Mr. A, who is riding on a rocket flying at a constant velocity, and Mr. B, who is observing Mr. A from a stationary space station. Assume that Mr. A is in the middle of the rocket. Sensors have been placed at the front and back of the rocket. Mr. A throws balls (or emits light) toward the front and back of the rocket. We will observe how those balls (or light beams) hit the sensors at the front and back of the rocket.

### CASE OF NEWTONIAN VELOCITY ADDITION (NONRELATIVISTIC ADDITION)

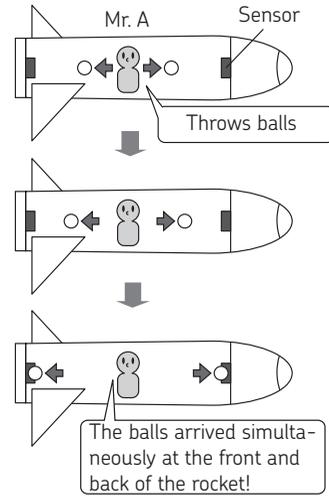
First, we will use the motion of the balls to consider the case in which velocities are added in a Newtonian mechanical manner (before considering relativity).

First, let’s look at Mr. A as shown in Figure 1–8. Since from Mr. A’s perspective the rocket is not moving, the balls, which are moving at the same velocity from the center toward the sensors at the front and back of the rocket, arrive at the sensors “simultaneously.”

Next, when observed by Mr. B from the space station, the rocket advances in the direction of travel. In other words, using the point of departure of the balls (dotted line) as a reference, the front of the ship moves away from the dotted line, and the back of the ship approaches the dotted line. However, since the velocity of the rocket is added to the velocity of the ball in the forward direction, according to normal addition, the ball’s velocity increases and it catches up with the front of the ship. On the other hand, the velocity of the ball toward the back of the ship is reduced by the velocity of the rocket (indicated by the short arrow in the figure), and the back of the ship catches up to the ball. Therefore, Mr. B also observes that the balls arrive at the front and back of the ship “simultaneously.”

Nonrelativistic addition:

Mr. A observes the motion of the balls inside the rocket.



Nonrelativistic addition:

Mr. B observes the motion of the balls inside the rocket from his space station. Since the balls are moving together with the rocket, the velocity of the ball is increased by the velocity of the rocket toward the front of the rocket and decreased by the velocity of the rocket toward the back of the rocket. Therefore, the balls arrive "simultaneously" (the lengths of the arrows indicate the difference in the velocities of the balls).

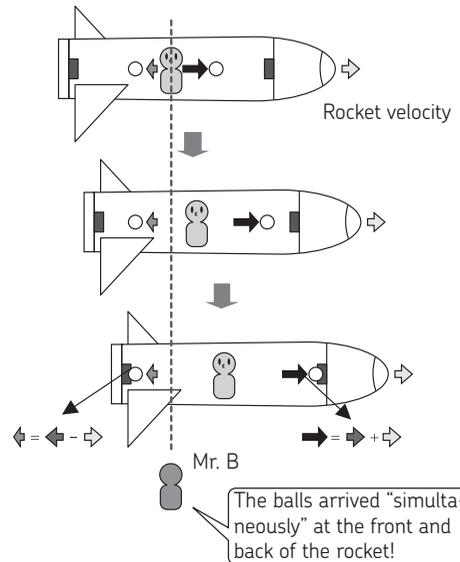


Figure 1-8: Newtonian velocity addition

## CASE IN WHICH THE SPEED OF LIGHT IS CONSTANT (RELATIVISTIC ADDITION OF VELOCITY)

Now let's consider the case in which the speed of light is constant. Instead of throwing balls, Mr. A will emit light while traveling at nearly the speed of light (see Figure 1-9).

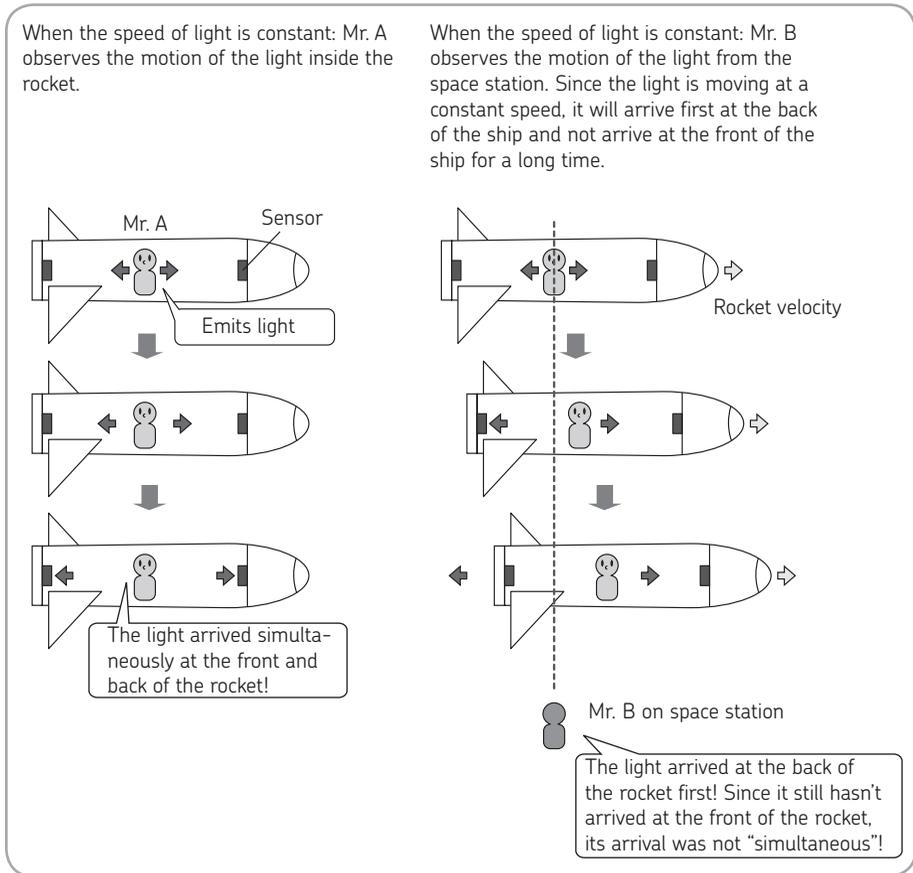


Figure 1-9: Case in which the speed of light is constant (relativistic addition of velocity)

You may have already realized what is at issue: Mr. B's observation will differ from that of Mr. A.

For Mr. A, even when the speed of light is constant, the light will arrive "simultaneously" at the front and back of the rocket.

However, when observed by Mr. B, the light moving towards the front of the ship does not arrive for a long time. It has to overtake the ship, which is moving away at nearly the speed of the light. Therefore, the light arrives at the back of the ship before it reaches the front of the ship.

That's right; when observed by Mr. B, the light does not arrive "simultaneously" at the front and back of the ship.

The simultaneity property of light differs in this way depending on the standpoint of the observer. This is called *simultaneity mismatch*.

# GALILEAN PRINCIPLE OF RELATIVITY AND GALILEAN TRANSFORMATION

The Galilean principle of relativity says that “the laws of physics are the same regardless of whether the coordinate system from which the observation is made is at rest or moving at a constant velocity.” In other words, Newtonian mechanics (the physical laws that govern motion) are always the same, regardless of whether observations are made in a reference frame that is at rest or one that is moving at a constant velocity. This principle was derived from an experiment in which an iron ball was dropped from the mast of a ship, as shown in Figure 1-10. The iron ball fell directly under the mast whether the ship was moving or at rest.

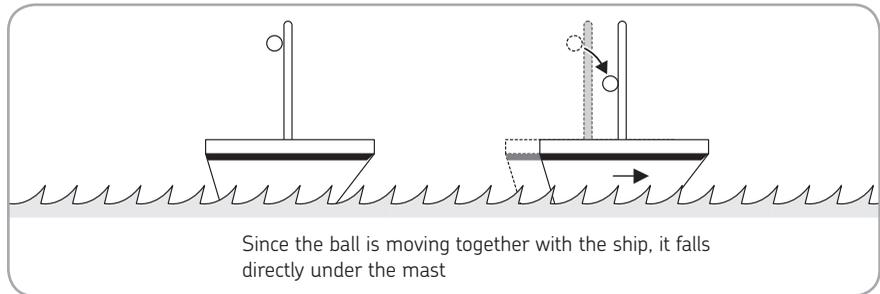


Figure 1-10: Galilean principle of relativity

Since the laws of physics are the same in any reference frame, Galileo arrived at a straightforward way to describe how observations look different depending on which reference frame you are in. Today we use algebraic equations called the *Galilean transformation* to help understand the notion of “adding” relative velocities.

Let’s take two coordinate systems, one with the coordinates  $(x, t)$  and the other with coordinates  $(x', t')$ , where  $x$  and  $x'$  describe position and  $t$  and  $t'$  describe time. One can go from one coordinate system to the other, by considering the relative velocity between the two coordinate systems  $v$ .

$$x' = x - vt$$

$$t' = t$$

The above equations show the relationship between coordinates from a coordinate system at rest and a coordinate system moving at a constant velocity  $v$  relative to the coordinate system at rest. Inertial frames are mutually linked in this way by the Galilean transformation. If we compare them using Newton’s equation of motion, we can prove that Newton’s equation of motion takes the same form in each inertial frame. In other words, when the Galilean principle of relativity holds, Newtonian mechanics will hold.

# DIFFERENCES BETWEEN THE GALILEAN PRINCIPLE OF RELATIVITY AND EINSTEIN'S SPECIAL PRINCIPLE OF RELATIVITY

As just described, the Galilean principle of relativity indicates that Newtonian mechanics apply across inertial frames when linked with the Galilean transformation.

On the other hand, the assumption that the speed of light is constant in any reference frame forced scientists to reformulate the Galilean transformation to be consistent with relativity. This new transformation is called the *Lorentz transformation*.

The Lorentz transformation is shown by the equations below, which show the relationship between coordinates from a coordinate system at rest and a coordinate system moving at a constant velocity  $v$  relative to the coordinate system at rest. The variables with the prime symbol ( $'$ ) attached represent coordinates observed from the coordinate system at rest; the variables without the prime symbol represent coordinates observed from the system in motion. Note that the speed of light  $c$  appears in the equations here. Another point to notice is that time  $t$  is transformed in a manner similar to that of length; time does not exist independently but must be considered to be unified with space.

$$x' = \frac{x - vt}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$
$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

## WAIT A SECOND—WHAT HAPPENS WITH THE ADDITION OF VELOCITIES?

When we assume that the speed of light is constant, what happens when velocities are added to the mix?

According to the principle of relativity, when calculated based on the Lorentz transformation, the addition of velocities is indicated by the following equation.

$$w = \frac{u + v}{1 + \frac{vu}{c^2}}$$

This equation describes the resulting addition of velocities of a missile  $w$  when the velocity of a rocket is  $v$  and the velocity (observed from the rocket) of the missile shot from the rocket is  $u$ , as shown in Figure 1-11. The difference is apparent when this equation is compared with the normal addition (nonrelativistic) equation  $w = u + v$ .

If we enter specific velocities in the above equations, we'll obtain some interesting results.

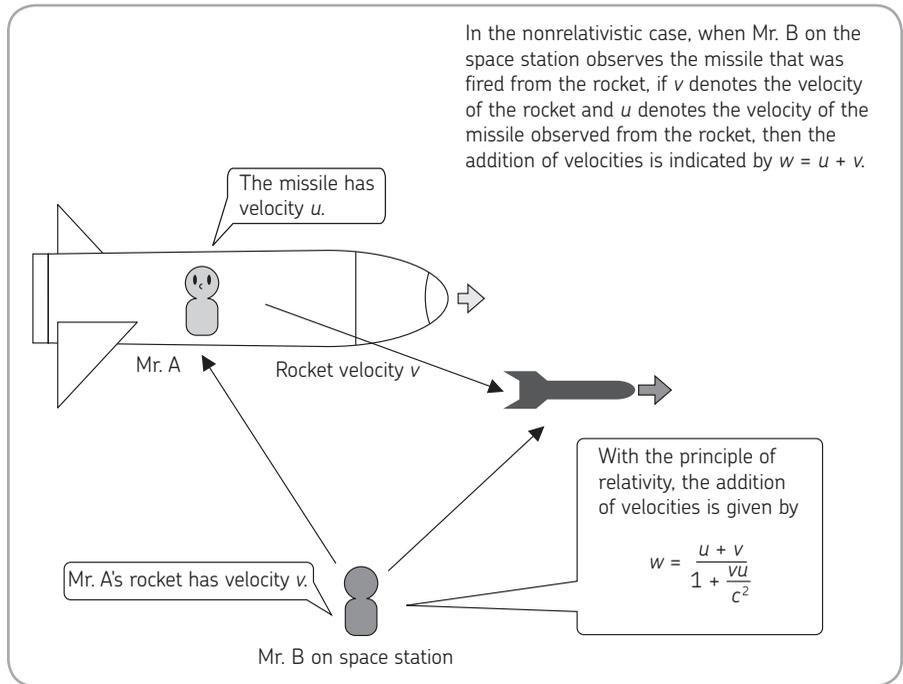


Figure 1-11: Addition of velocities

For example, when the rocket velocity  $v$  is 50 percent of the speed of light ( $0.5c$ ) and the missile velocity  $u$  observed from the rocket is 50 percent of the speed of light (also  $0.5c$ ), then the missile velocity  $w$  observed by Mr. B will be 80 percent of the speed of light ( $0.8c$ ).

$$w = \frac{(0.5c + 0.5c)}{\left(1 + \frac{(0.5c)^2}{c^2}\right)} = \frac{c}{1.25} = 0.8c$$

This equation also yields an interesting result when  $v$  and  $u$  are their maximum values. If the rocket velocity  $v$  is 100 percent of the speed of light (practically speaking,  $v = c$  is impossible for an object with mass, like a rocket) and the missile velocity  $u$  observed from the rocket is 100 percent of the speed of light, then the missile velocity  $w$  observed by Mr. B will be the speed of light.

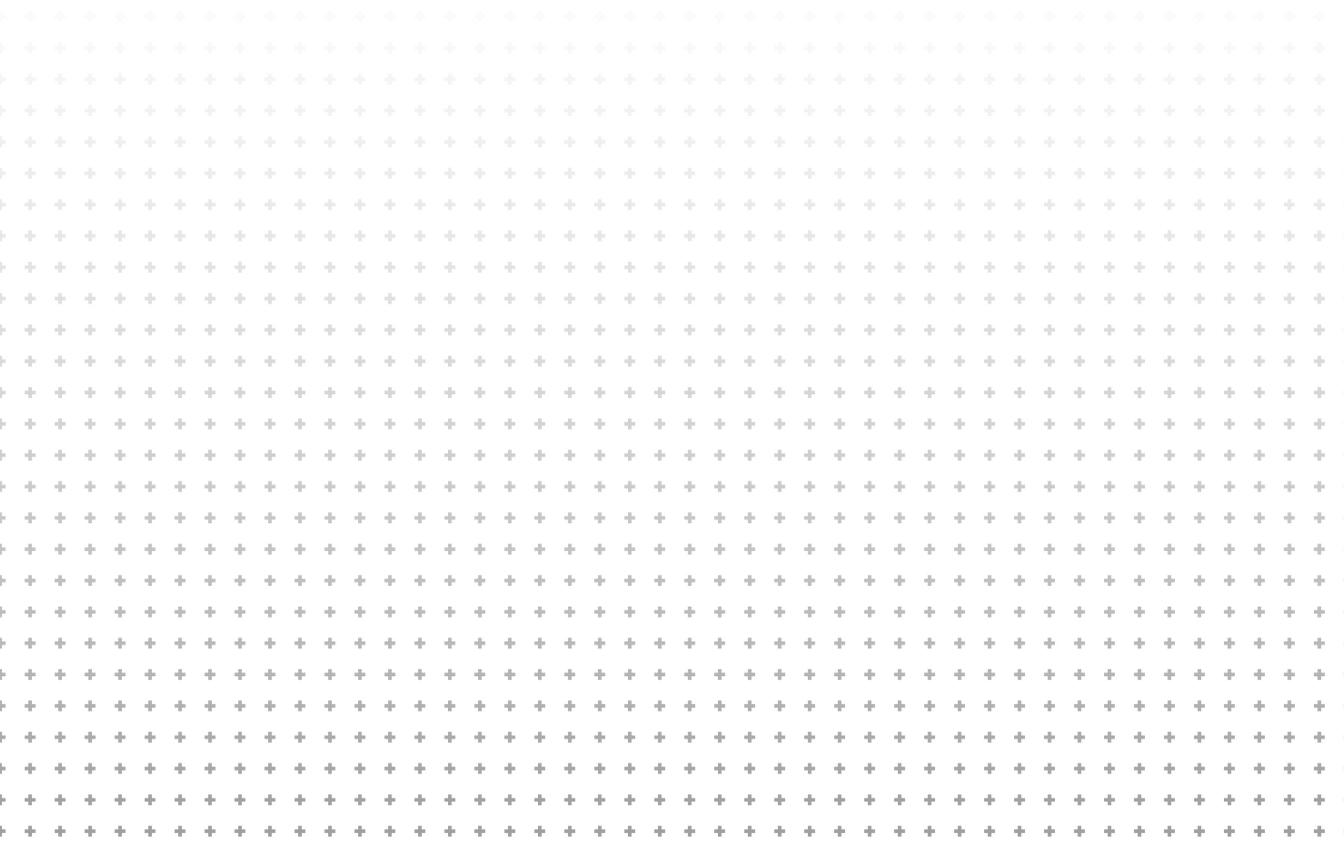
$$w = \frac{(c + c)}{\left(1 + \frac{c^2}{c^2}\right)} = \frac{2c}{2} = c$$

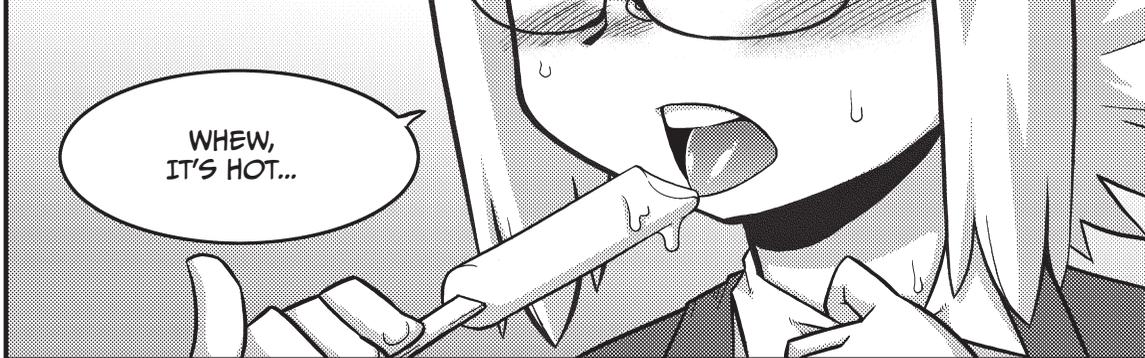
The speed of light cannot be exceeded under any circumstances!





WHAT DO YOU MEAN,  
TIME SLOWS DOWN?





WHEW,  
IT'S HOT...



YOU'RE A REAL PIECE OF WORK, USING A STUDENT AS A PERSONAL SERVANT TO GO AND BUY YOU A POPSICLE.

WHEW

HEY, YOU ALSO BOUGHT YOURSELF MONAKA WITH MY MONEY, DIDN'T YOU!



ANYWAY, DON'T COMPLAIN WHEN I'M TEACHING YOU FREE OF CHARGE! AND YOU TOOK YOUR TIME, MR. TURTLE!

HOW CAN YOU TALK THAT WAY TO YOUR STUDENT...



I GUESS IT'S OKAY SINCE I WAS ABLE TO GET MY OWN SNACK... IT'S GETTING HOTTER OUTSIDE.

THAT'S RIGHT. SPEAKING OF TURTLES, YOU KNOW THE STORY OF URISHAMA TARO, DON'T YOU?

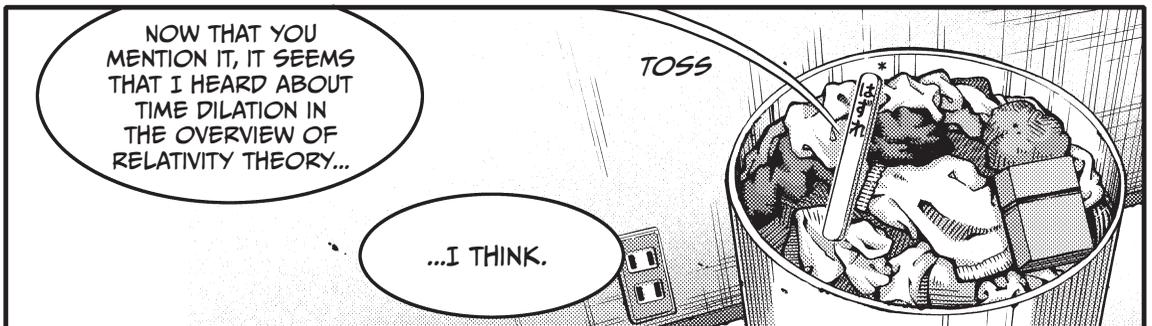
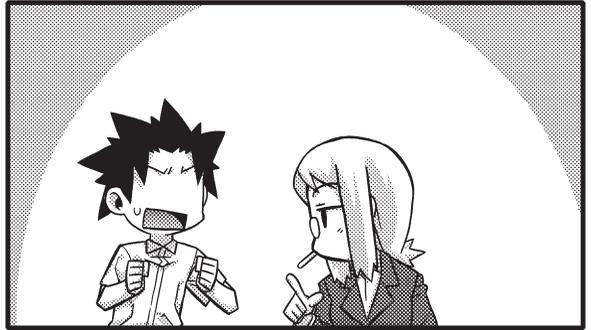
HUH?  
I KNOW THAT...



URASHIMA TARO IS A GUY WHO RESCUED A TURTLE AND WENT TO THE PALACE OF THE DRAGON GOD UNDER THE SEA.

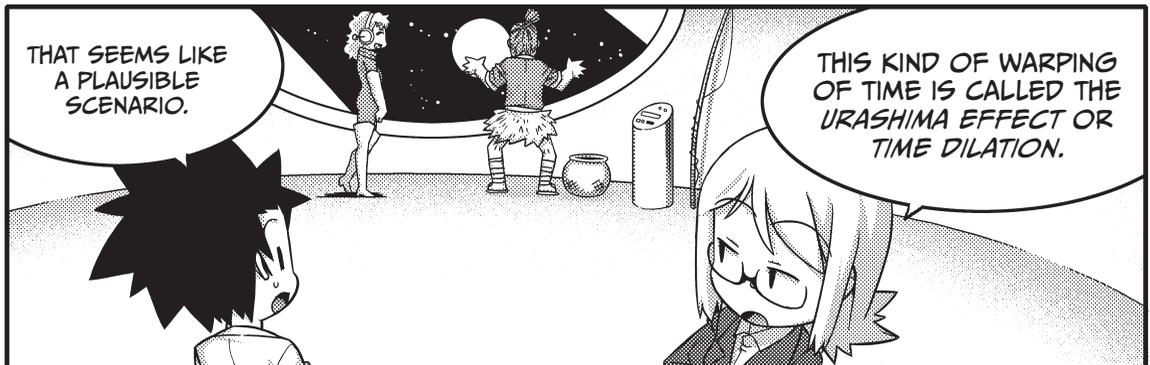
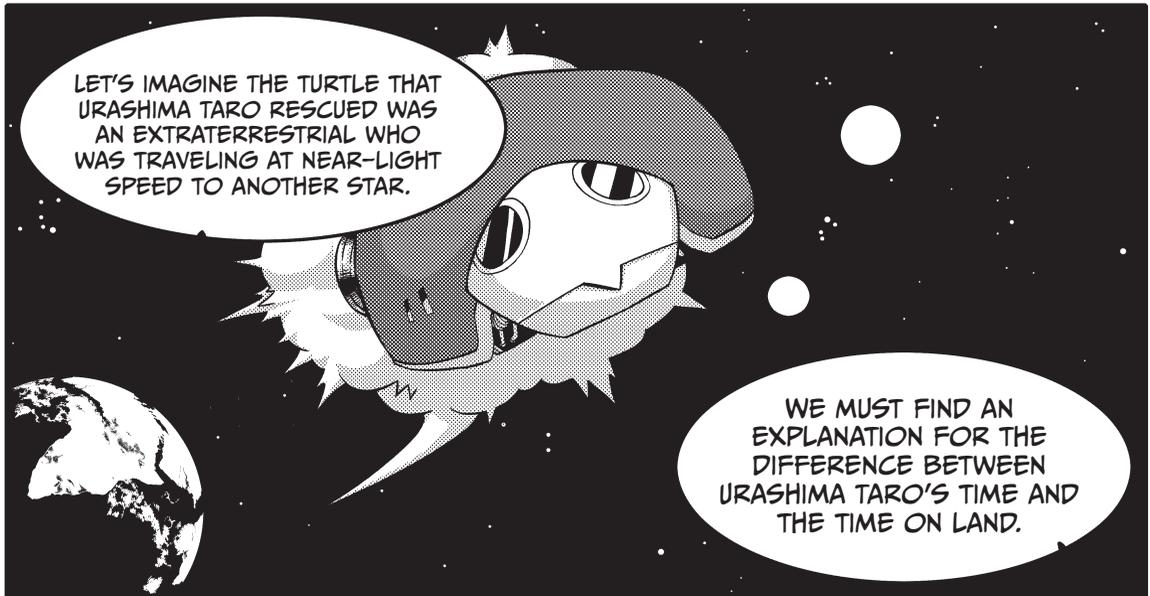
WHEN HE RETURNED HOME AFTER PLAYING THERE FOR A FEW DAYS, HE FOUND THAT SEVERAL HUNDRED YEARS HAD PASSED ON LAND, RIGHT?

STOP!



\* FAILURE  
TRANSLATOR'S NOTE: IN JAPAN, POPSICLE STICKS HAVE FORTUNES ON THEM.

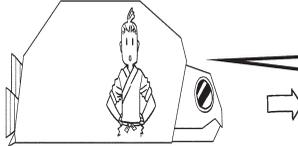
## 1. URASHIMA EFFECT (TIME DILATION)



ASSUME, FOR EXAMPLE, THAT A 10-TON SPACESHIP WITH A TOTAL LENGTH OF 100 METERS IS FLYING AT 99.6% OF THE SPEED OF LIGHT.

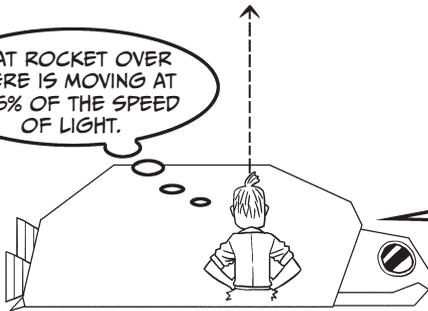
AN OBSERVER WILL PERCEIVE THE LENGTH CONTRACTING TO 9 METERS AND THE MASS INCREASING TO 110 TONS.

SPACE EXPLORER ON A ROCKET MOVING AT 99.6% OF THE SPEED OF LIGHT



THE SPACE EXPLORER'S ROCKET, WHICH IS MOVING AT 99.6% THE SPEED OF LIGHT AWAY FROM THE HOMEBODY'S ROCKET, IS OBSERVED BY THE HOMEBODY TO HAVE A LENGTH OF 9 M AND A MASS OF 110 T.

THAT ROCKET OVER THERE IS MOVING AT 99.6% OF THE SPEED OF LIGHT.

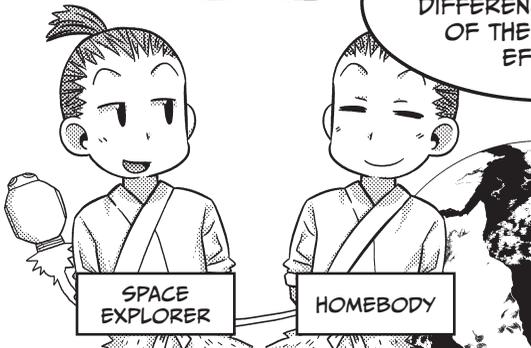


AS OBSERVED BY THE HOMEBODY, HIS OWN ROCKET HAS A LENGTH OF 100 M AND A MASS OF 10 T.

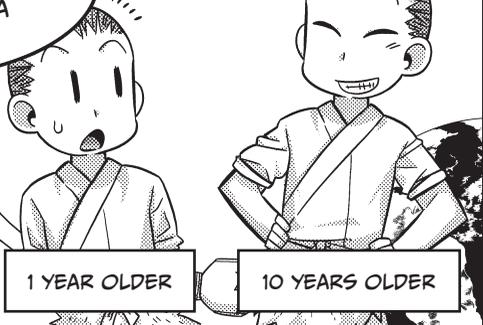
HOMEBODY ON A ROCKET AT REST

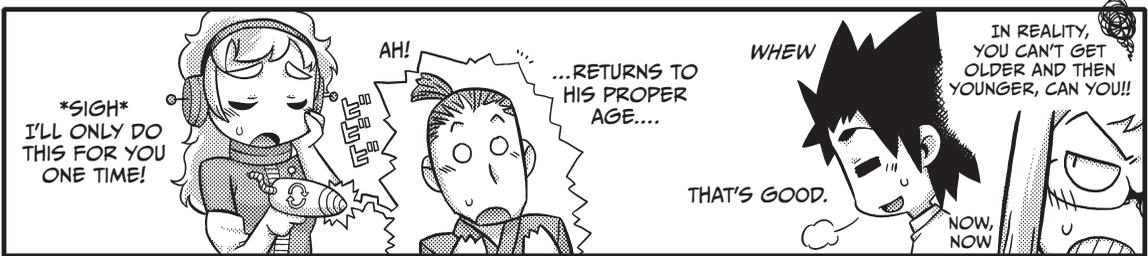
AND IF IT TAKES OFF FROM EARTH, TRAVELS FOR 1 YEAR, AND THEN RETURNS, APPROXIMATELY 10 YEARS WILL HAVE PASSED ON EARTH.

THE SAME AGE



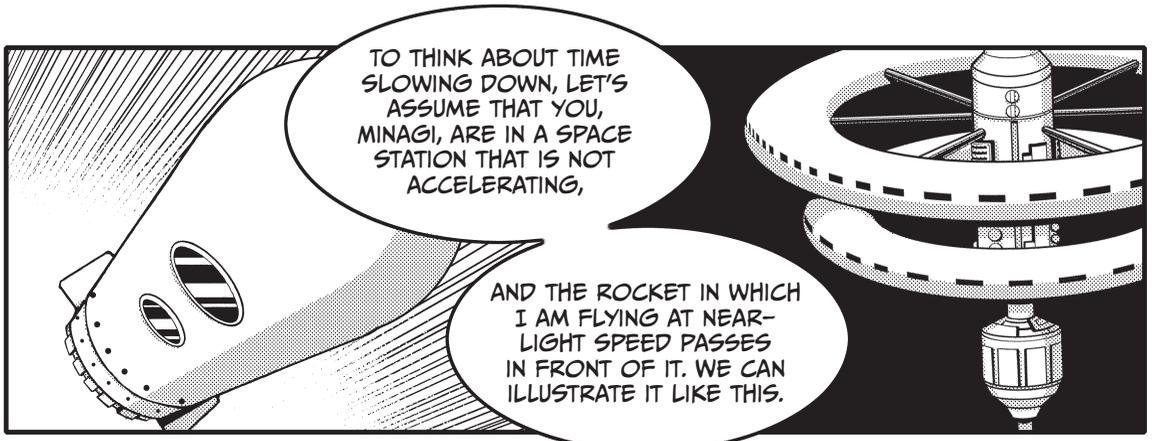
IN OTHER WORDS, THERE IS A 9-YEAR AGE DIFFERENCE BECAUSE OF THE URASHIMA EFFECT.





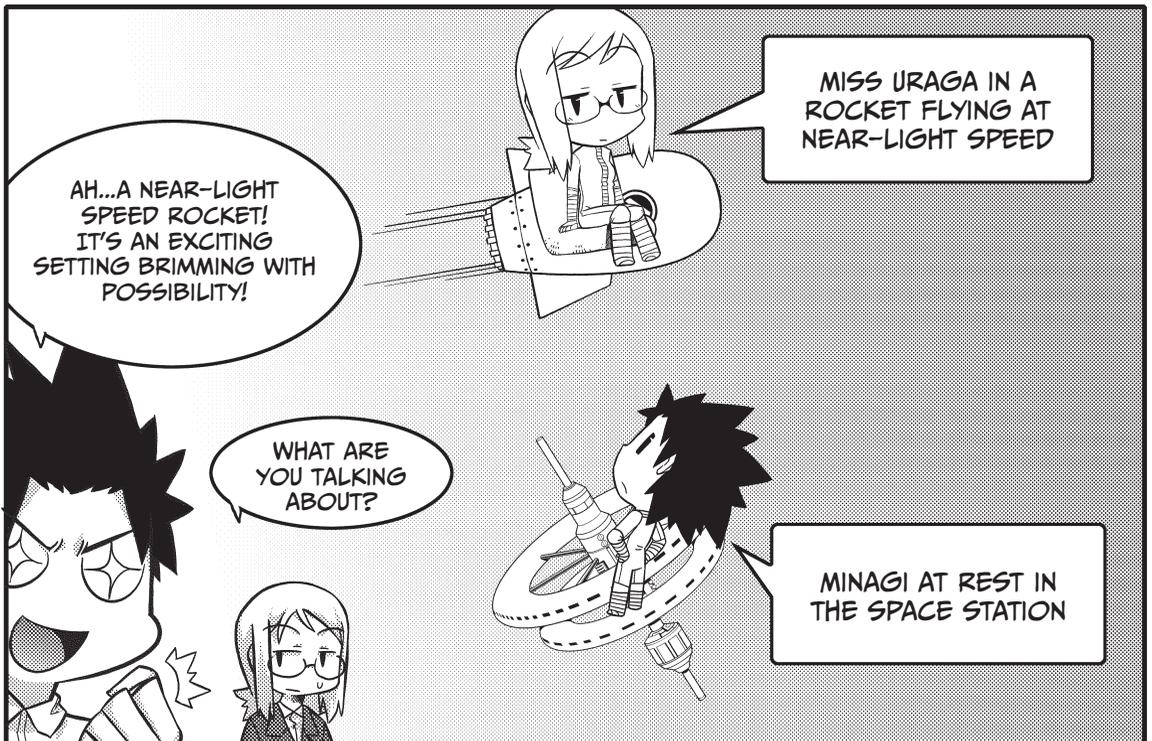
2. WHY DOES TIME SLOW DOWN?





TO THINK ABOUT TIME SLOWING DOWN, LET'S ASSUME THAT YOU, MINAGI, ARE IN A SPACE STATION THAT IS NOT ACCELERATING,

AND THE ROCKET IN WHICH I AM FLYING AT NEAR-LIGHT SPEED PASSES IN FRONT OF IT. WE CAN ILLUSTRATE IT LIKE THIS.



AH...A NEAR-LIGHT SPEED ROCKET! IT'S AN EXCITING SETTING BRIMMING WITH POSSIBILITY!

MISS URAGA IN A ROCKET FLYING AT NEAR-LIGHT SPEED

WHAT ARE YOU TALKING ABOUT?

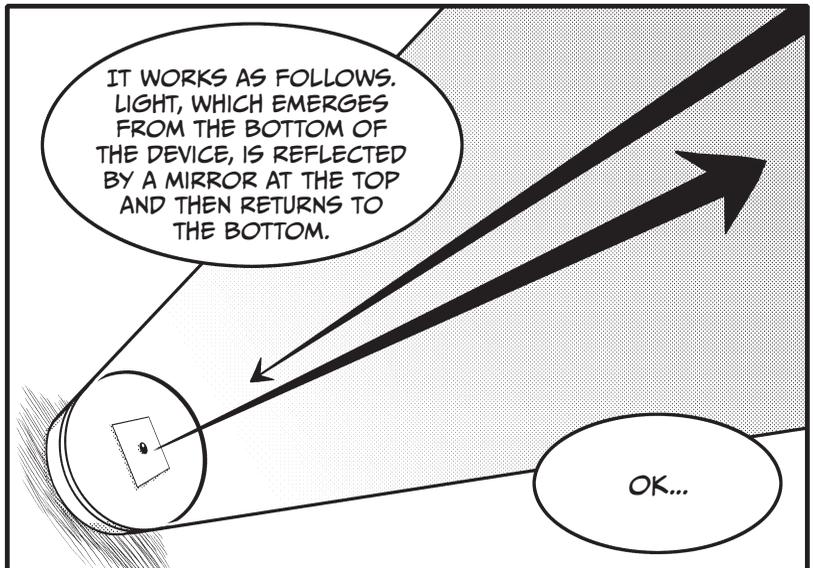
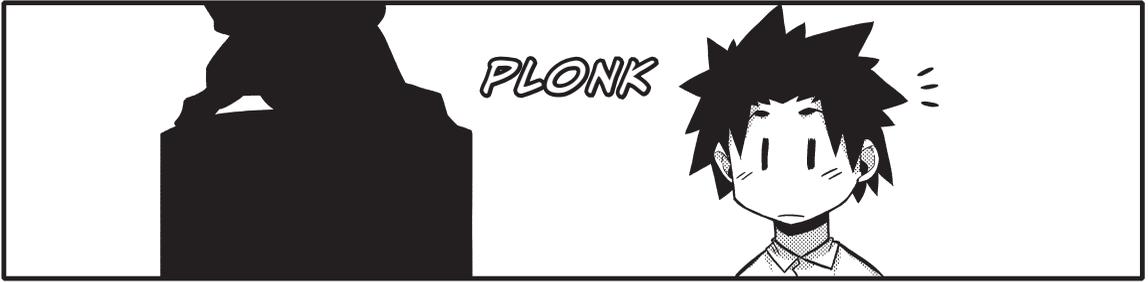
MINAGI AT REST IN THE SPACE STATION



IN THIS SETTING, WE WILL MEASURE THEIR RESPECTIVE TIMES.

HOWEVER!  
TO MEASURE TIME HERE, WE WILL USE A SPECIAL DEVICE THAT MAKES USE OF THE PRINCIPLE THAT THE SPEED OF LIGHT IS CONSTANT.

SPECIAL DEVICE?

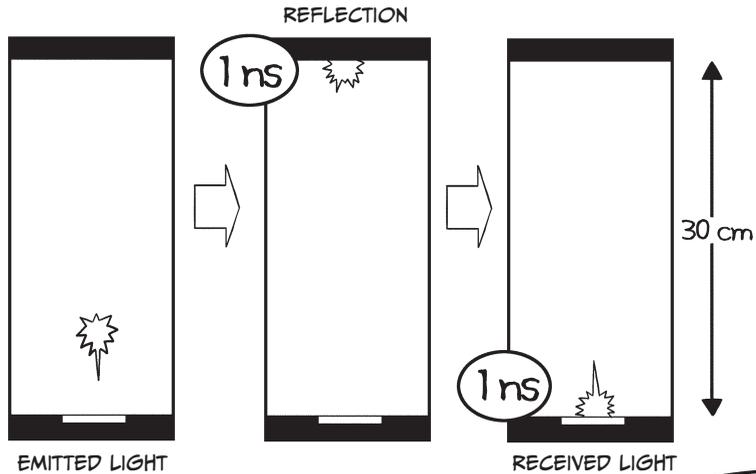


SINCE THE LENGTH OF THE TUBE IS 30 CENTIMETERS...IT TAKES 1 ns (NANOSECOND) FOR THE LIGHT THAT WAS EMITTED TO REACH THE TOP...

AND WHEN IT IS REFLECTED AND RETURNS TO THE BOTTOM, ANOTHER 1 ns IS ALSO COUNTED. BY THE WAY, 1 ns MEANS 1 BILLIONTH OF A SECOND.



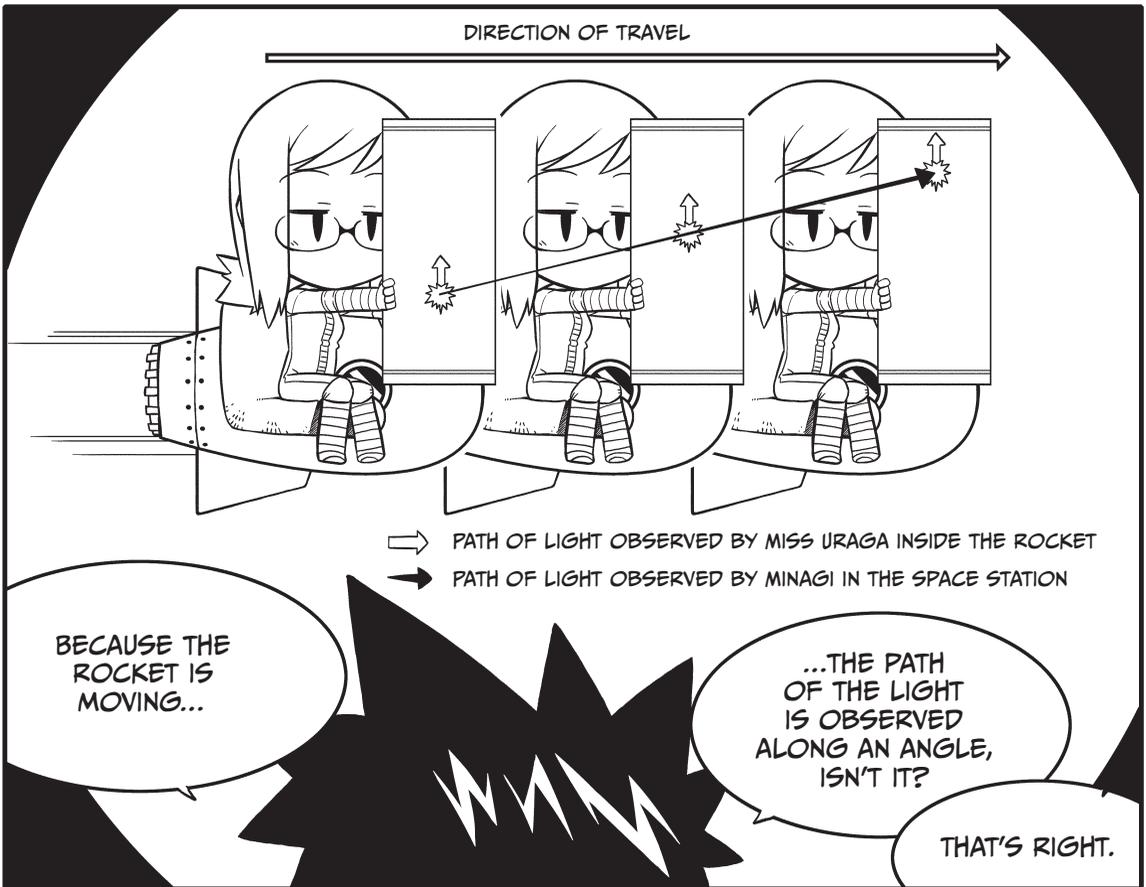
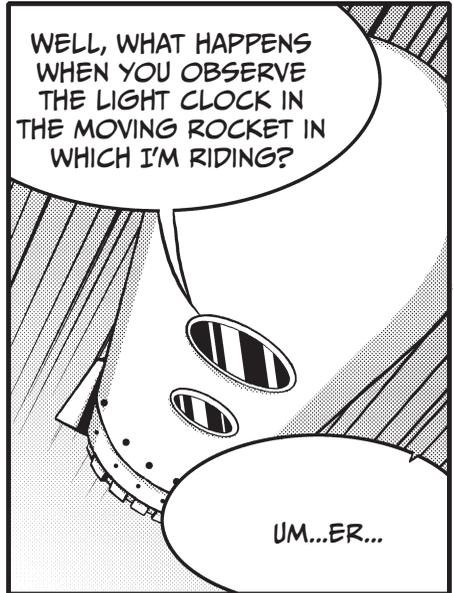
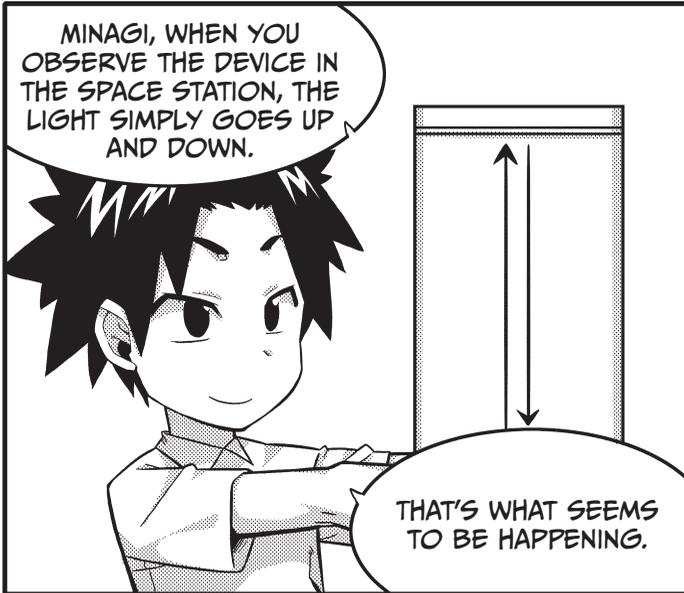
FLOW OF TIME



TIME IS BEING MEASURED BY THE MOVEMENT OF LIGHT, RIGHT?

MINAGI (IN THE SPACE STATION) AND I (RIDING IN THE ROCKET) EACH HAVE ONE OF THESE CLOCKS, AND WE WILL TRY CHECKING HOW TIME IS PROGRESSING FOR EACH OF US.

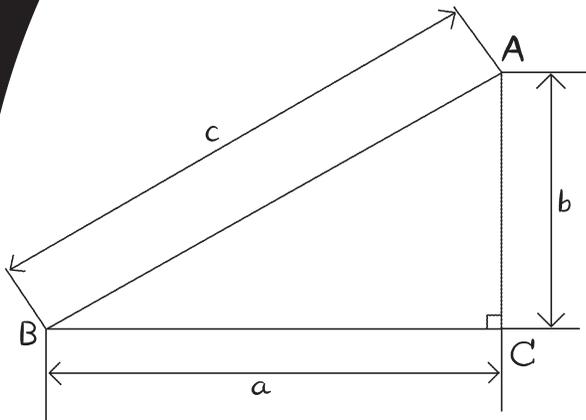
I SEE.



MINAGI OBSERVES LIGHT IN HIS CLOCK MOVE DIRECTLY UPWARDS AND DOWNWARDS, BUT MINAGI OBSERVES THE LIGHT IN MY CLOCK PROCEED AT A LONGER PATH, AT AN ANGLE.



LET'S CONSIDER THIS USING THE PYTHAGOREAN THEOREM.



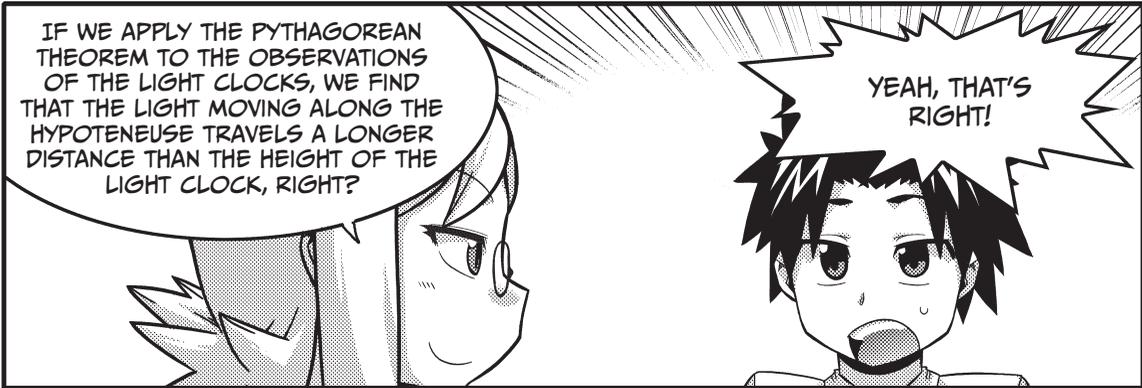
PYTHAGOREAN THEOREM

$$c^2 = a^2 + b^2$$

THE PYTHAGOREAN THEOREM STATES THAT "THE SQUARE OF THE LENGTH OF THE HYPOTENUSE IS EQUAL TO THE SUM OF THE SQUARES OF THE OTHER TWO SIDES"...THAT IS...

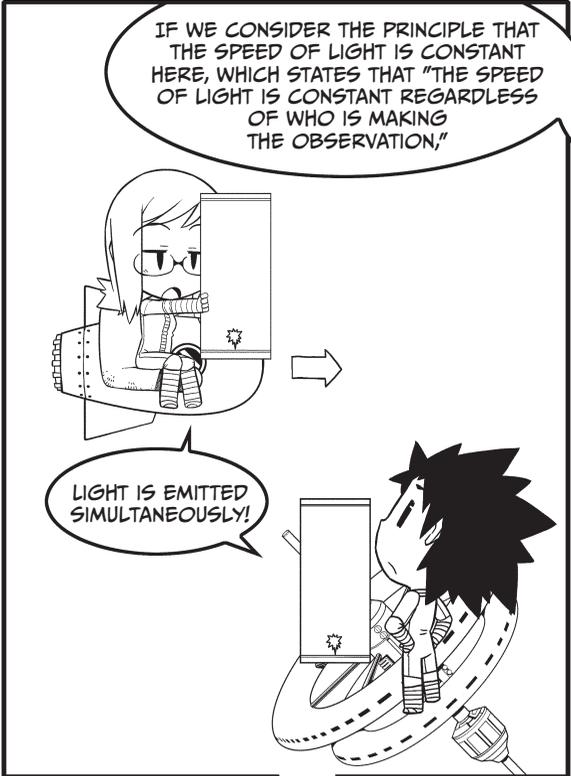
IT'S THE THEOREM THAT SAYS FOR THIS RIGHT TRIANGLE ABC, THE RELATIONSHIP  $c^2 = a^2 + b^2$  HOLDS, RIGHT?





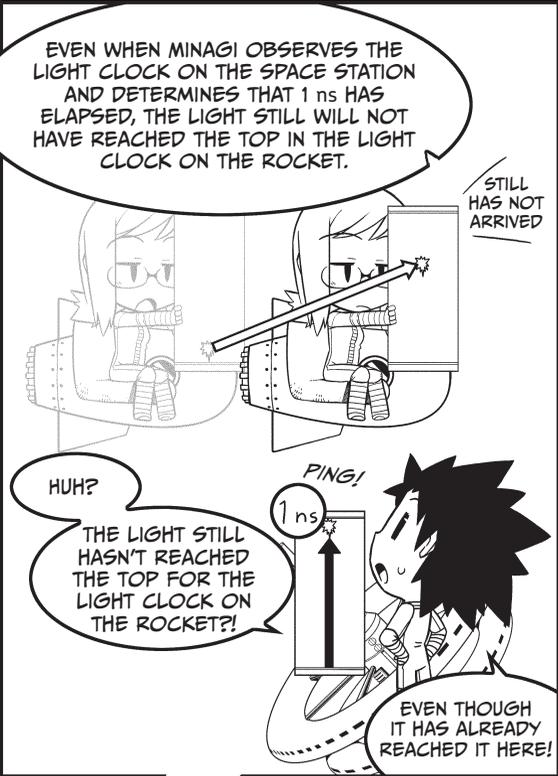
IF WE APPLY THE PYTHAGOREAN THEOREM TO THE OBSERVATIONS OF THE LIGHT CLOCKS, WE FIND THAT THE LIGHT MOVING ALONG THE HYPOTENEUSE TRAVELS A LONGER DISTANCE THAN THE HEIGHT OF THE LIGHT CLOCK, RIGHT?

YEAH, THAT'S RIGHT!



IF WE CONSIDER THE PRINCIPLE THAT THE SPEED OF LIGHT IS CONSTANT HERE, WHICH STATES THAT "THE SPEED OF LIGHT IS CONSTANT REGARDLESS OF WHO IS MAKING THE OBSERVATION,"

LIGHT IS EMITTED SIMULTANEOUSLY!



EVEN WHEN MINAGI OBSERVES THE LIGHT CLOCK ON THE SPACE STATION AND DETERMINES THAT 1 ns HAS ELAPSED, THE LIGHT STILL WILL NOT HAVE REACHED THE TOP IN THE LIGHT CLOCK ON THE ROCKET.

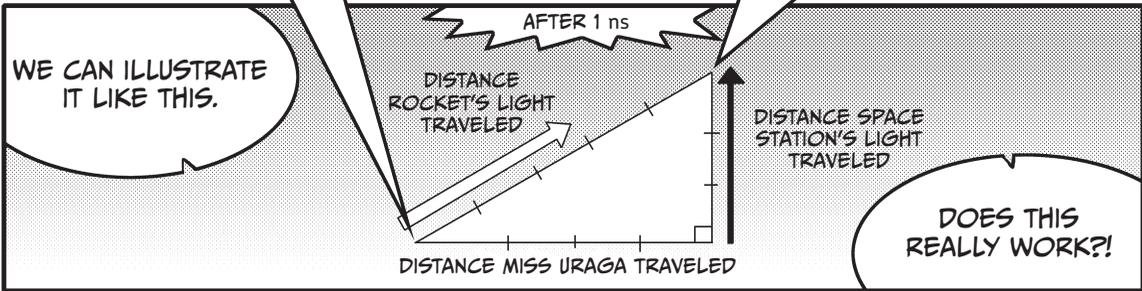
STILL HAS NOT ARRIVED

HUH?

THE LIGHT STILL HASN'T REACHED THE TOP FOR THE LIGHT CLOCK ON THE ROCKET?!

PING!  
1 ns

EVEN THOUGH IT HAS ALREADY REACHED IT HERE!



WE CAN ILLUSTRATE IT LIKE THIS.

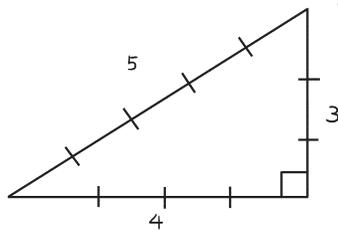
AFTER 1 ns  
DISTANCE ROCKET'S LIGHT TRAVELED  
DISTANCE SPACE STATION'S LIGHT TRAVELED  
DISTANCE MISS URAGA TRAVELED

DOES THIS REALLY WORK?!

WHEN THE LIGHT IN THE ROCKET'S CLOCK IS OBSERVED BY MINAGI TO HAVE RETURNED TO THE BOTTOM, MORE THAN 2 ns WILL HAVE ELAPSED ON MINAGI'S CLOCK.

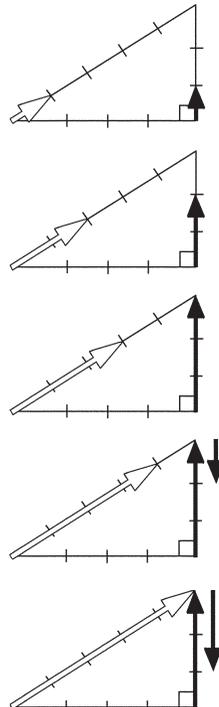
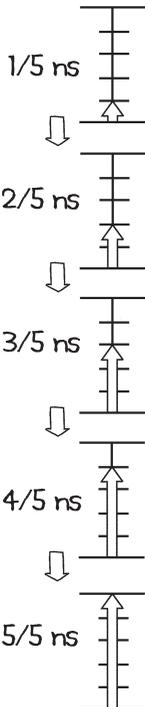
IN OTHER WORDS, TIME ADVANCES SLOWER FOR THE ROCKET I'M RIDING ON.

ALTHOUGH THIS STILL SOUNDS STRANGE, IT'S WHAT ACTUALLY HAPPENS.



WHEN A 3:4:5 RIGHT TRIANGLE IS USED AS AN EXAMPLE

TIME ELAPSED (AND LIGHT MOVEMENT) OBSERVED FROM MINAGI ON THE SPACE STATION



LIGHT ON THE SPACE STATION (OBSERVED BY MINAGI)



LIGHT ON THE ROCKET (OBSERVED BY MINAGI)



1/3 ns

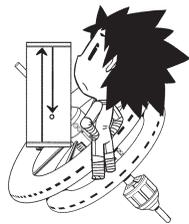
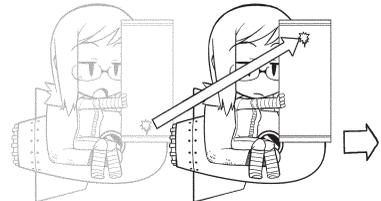
2/3 ns

3/3 ns

4/3 ns

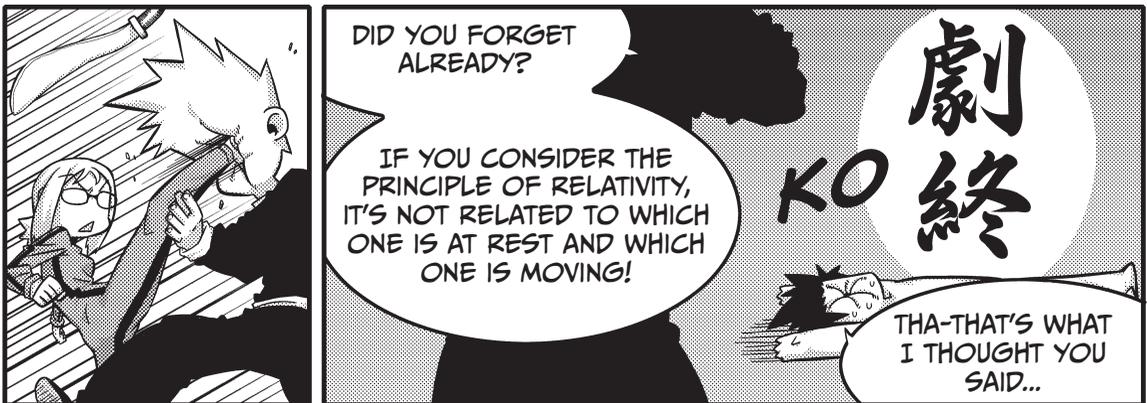
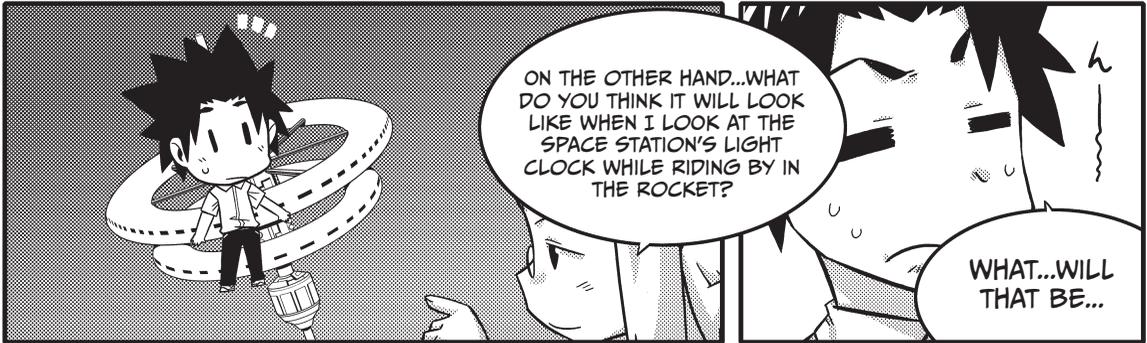
5/3 ns

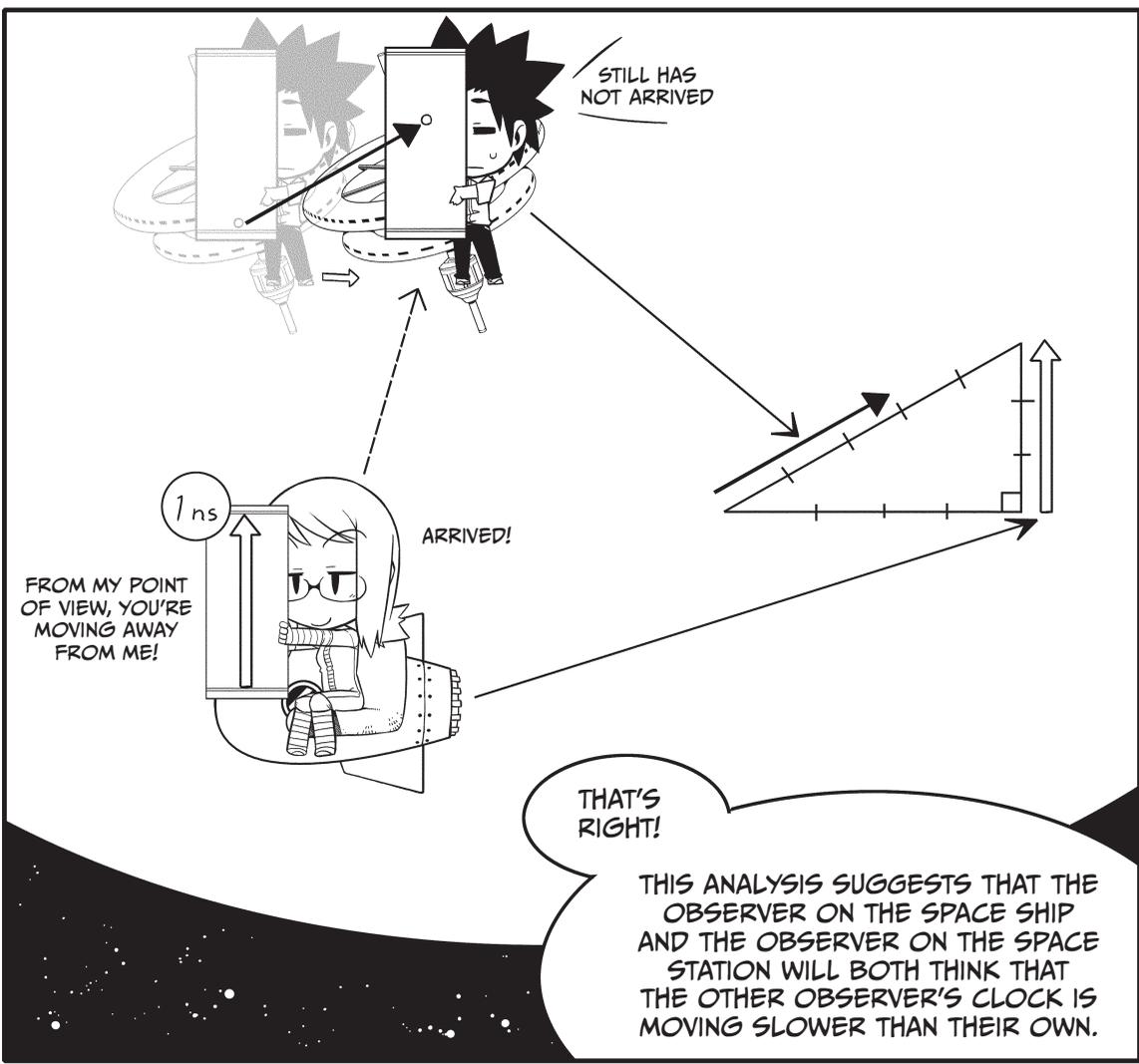
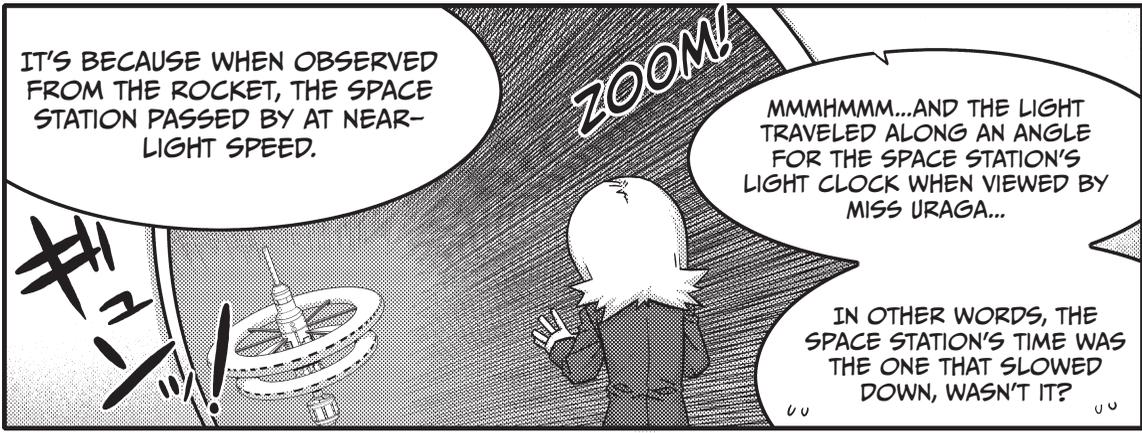
THE PATH OF LIGHT INSIDE THE ROCKET, AS OBSERVED BY THE SPACE STATION



THE PATH OF LIGHT INSIDE THE SPACE STATION, AS OBSERVED BY THE SPACE STATION

3. THE SLOWING OF TIME MUTUALLY AFFECTS EACH PARTY EQUALLY



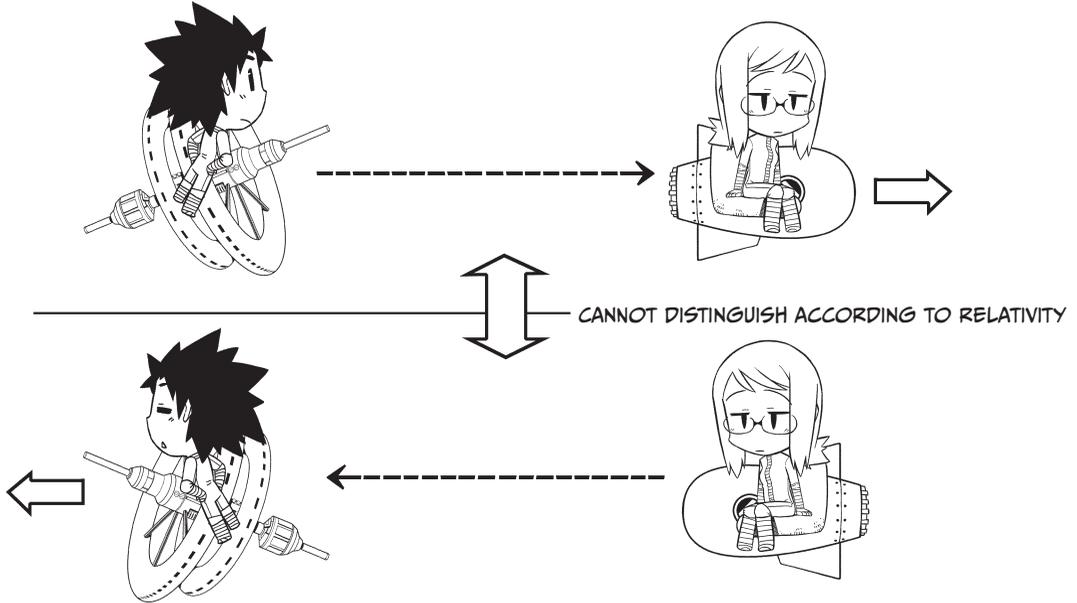


HUH?

JUST WAIT A SECOND!  
...WHAT WAS THAT?

WHAT'S THE  
MATTER?

WHEN OBSERVED FROM THE SPACE STATION, THE ROCKET IS MOVING.



WHEN OBSERVED FROM THE ROCKET, THE SPACE STATION IS MOVING.

THIS MEANS THE  
URASHIMA EFFECT DOES  
NOT OCCUR, DOESN'T IT?!  
IT'S A LIE! IT'S A FRAUD!!

WHEN THE OBSERVER ON THE SPACE  
STATION AND THE OBSERVER ON  
THE ROCKET SHIP MEET AFTER THE  
ROCKET SHIP RETURNS, BOTH OF  
THEM CAN'T THINK THAT THE OTHER  
PERSON HAS AGED SLOWER.  
ONE OF THEM MUST BE RIGHT  
AND THE OTHER WRONG.

FRAUD?!

THAT'S RIGHT!  
IT'S STRANGE, ISN'T IT?!

THIS IS THE FAMOUS  
THOUGHT EXPERIMENT  
KNOWN AS THE TWIN  
PARADOX.



TWIN...?

TWINS

OLDER  
SISTER

YOUNGER  
SISTER

FOR EXAMPLE, ASSUME  
THERE ARE TWIN SISTERS,  
AND THE YOUNGER SISTER  
(BY A FEW MINUTES)  
REMAINS ON EARTH...

WHILE THE OLDER  
SISTER TRAVELS  
THROUGH SPACE AT  
NEAR-LIGHT SPEED.

BRING  
ME A  
SOUVENIR!

DON'T  
WORRY  
ABOUT ME!

BE  
CAREFUL,  
OK?

ACCORDING TO THE  
SPECIAL THEORY OF  
RELATIVITY, TIME FLOWS  
MORE SLOWLY FOR THE  
OLDER SISTER WHO MOVES  
AT NEAR-LIGHT SPEED...

I'M BACK! ♪

I BROUGHT YOU VENUSIAN  
CANNED CRAB AND ALSO  
A SPACE BEAR WOOD  
CARVING.

AND WHEN SHE  
RETURNS TO EARTH,  
THE YOUNGER SISTER  
HAS BECOME OLDER.

MY DEAR OLDER  
SISTER...YOU'VE  
STAYED SO  
YOUNG!

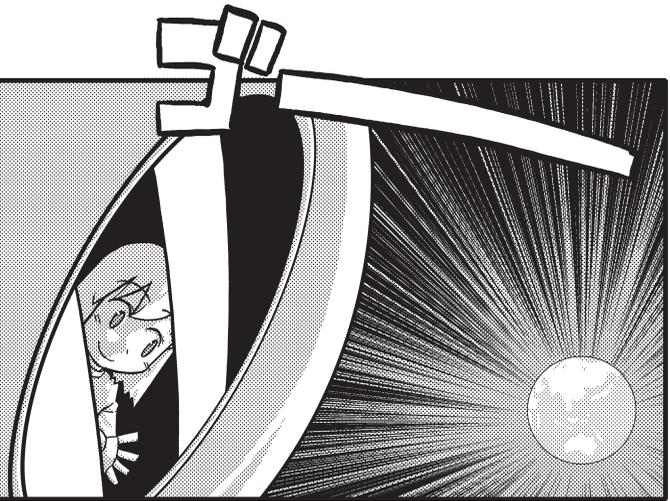
DO YOU HAVE  
ANYTHING ELSE?  
THOSE SOUVENIRS  
SOUND AWFUL!

WOW...YOU'VE  
REALLY GROWN UP,  
HAVEN'T YOU?

THIS IS THE  
URASHIMA EFFECT,  
RIGHT?

BUT FROM THE OLDER SISTER'S STANDPOINT, EARTH, WHERE THE YOUNGER SISTER IS, SEEMS TO MOVE AWAY AT NEAR-LIGHT SPEED. THE YOUNGER SISTER'S TIME SLOWS DOWN. THERE SEEMS TO BE A CONTRADICTION.

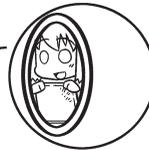
I SEE WHAT YOU'RE SAYING...



THE YOUNGER SISTER CLAIMS, "MY OLDER SISTER'S TIME SLOWS DOWN" SINCE THE ROCKET IS MOVING.



BOTH OF THEM MUTUALLY CLAIM THAT THE OTHER TWIN'S "TIME SLOWS DOWN." THIS IS A PARADOX.



THE OLDER SISTER CLAIMS, "MY YOUNGER SISTER'S TIME SLOWS DOWN" SINCE EARTH IS MOVING.

ACCORDING TO THE THEORY, TIME SLOWS DOWN FOR THE OLDER SISTER, WHO HAD RIDDEN IN THE ROCKET, AND WHEN SHE RETURNS TO EARTH, THE YOUNGER SISTER WILL BE OLDER.

THIS CRAB IS UNEXPECTEDLY DELICIOUS.

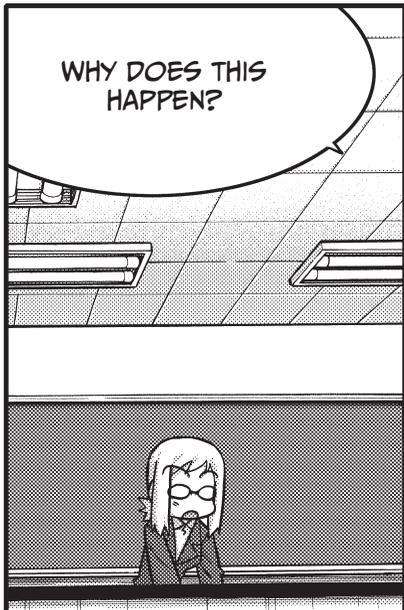


IT SURE IS, ISN'T IT?

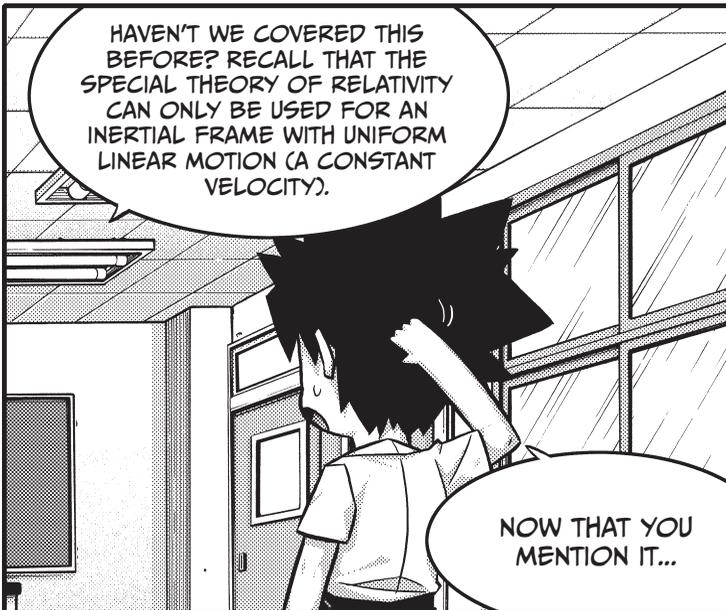


CAN I HAVE SOME?



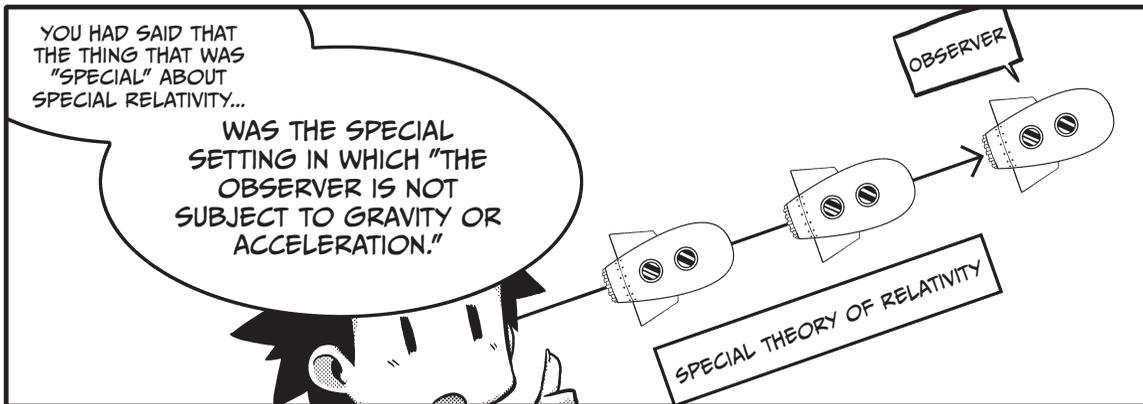


WHY DOES THIS HAPPEN?



HAVEN'T WE COVERED THIS BEFORE? RECALL THAT THE SPECIAL THEORY OF RELATIVITY CAN ONLY BE USED FOR AN INERTIAL FRAME WITH UNIFORM LINEAR MOTION (A CONSTANT VELOCITY).

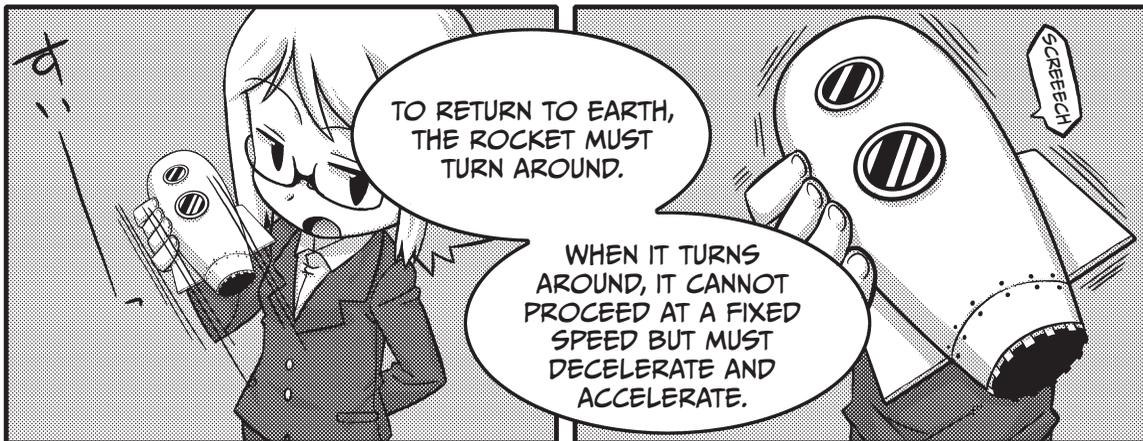
NOW THAT YOU MENTION IT...



YOU HAD SAID THAT THE THING THAT WAS "SPECIAL" ABOUT SPECIAL RELATIVITY...

WAS THE SPECIAL SETTING IN WHICH "THE OBSERVER IS NOT SUBJECT TO GRAVITY OR ACCELERATION."

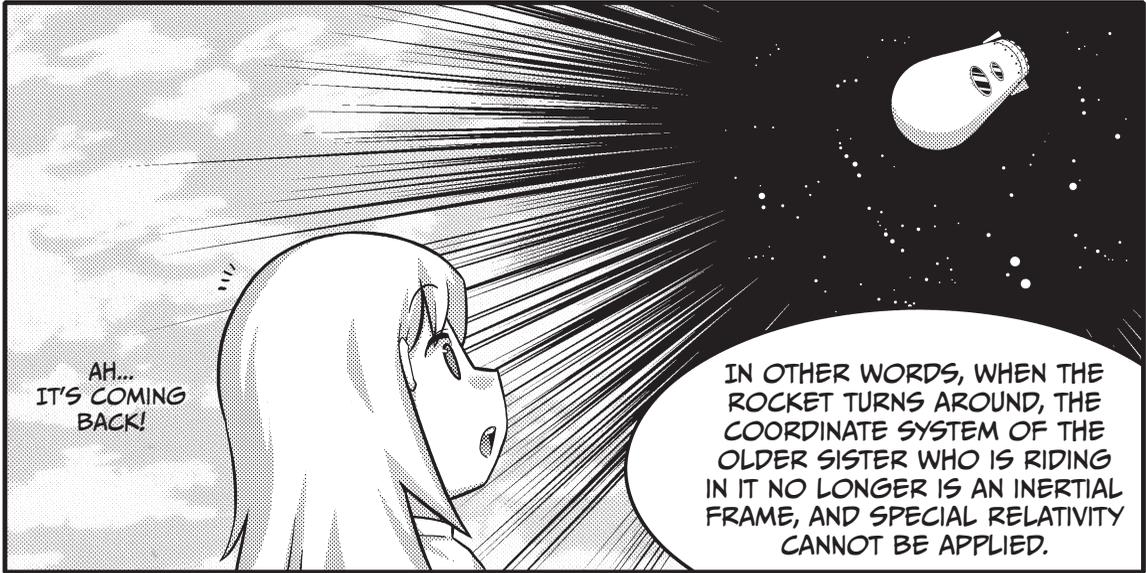
SPECIAL THEORY OF RELATIVITY



TO RETURN TO EARTH, THE ROCKET MUST TURN AROUND.

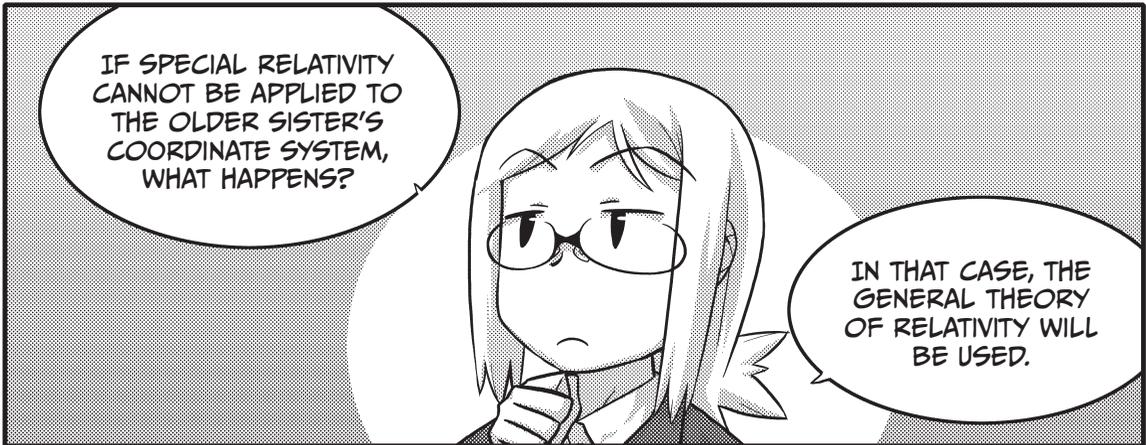
WHEN IT TURNS AROUND, IT CANNOT PROCEED AT A FIXED SPEED BUT MUST DECELERATE AND ACCELERATE.

SCREECH



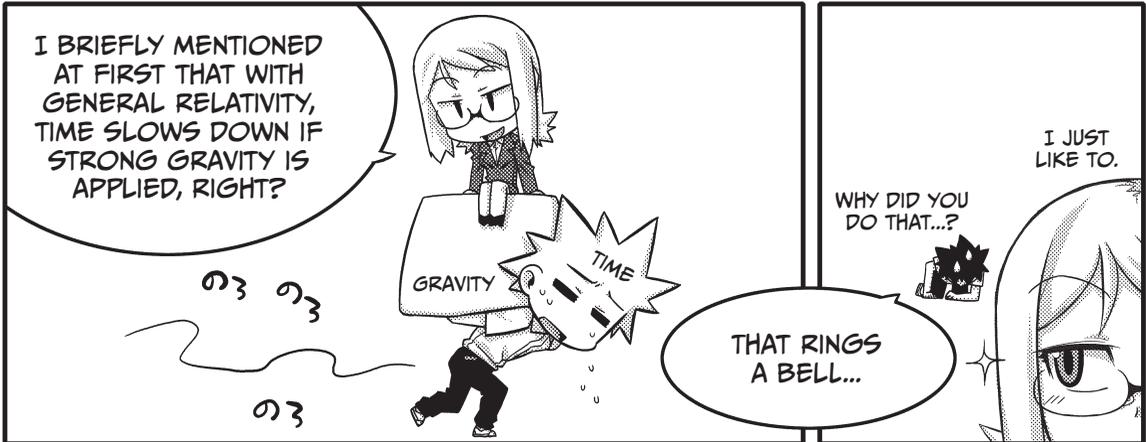
AH...  
IT'S COMING  
BACK!

IN OTHER WORDS, WHEN THE  
ROCKET TURNS AROUND, THE  
COORDINATE SYSTEM OF THE  
OLDER SISTER WHO IS RIDING  
IN IT NO LONGER IS AN INERTIAL  
FRAME, AND SPECIAL RELATIVITY  
CANNOT BE APPLIED.



IF SPECIAL RELATIVITY  
CANNOT BE APPLIED TO  
THE OLDER SISTER'S  
COORDINATE SYSTEM,  
WHAT HAPPENS?

IN THAT CASE, THE  
GENERAL THEORY  
OF RELATIVITY WILL  
BE USED.

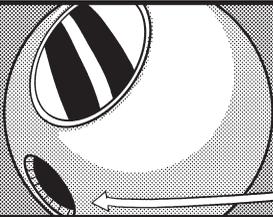


I BRIEFLY MENTIONED  
AT FIRST THAT WITH  
GENERAL RELATIVITY,  
TIME SLOWS DOWN IF  
STRONG GRAVITY IS  
APPLIED, RIGHT?

I JUST  
LIKE TO.  
WHY DID YOU  
DO THAT...?

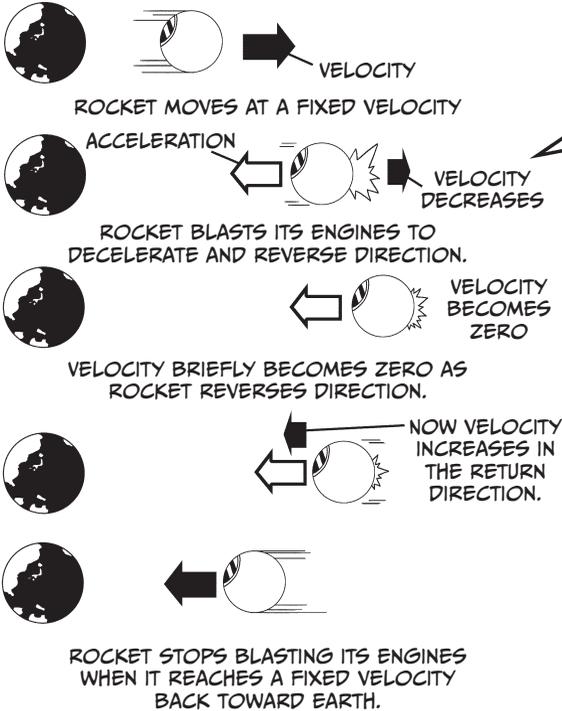
THAT RINGS  
A BELL...

TO MAKE IT EASIER TO UNDERSTAND, LET'S PREPARE THIS KIND OF ROCKET.

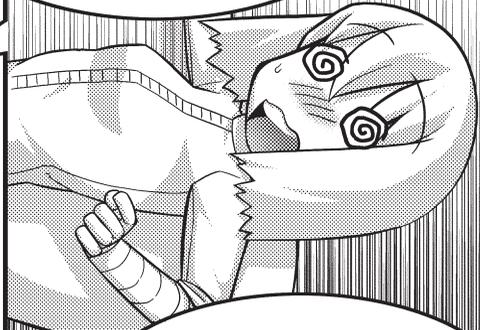


IT'S PERFECTLY ROUND AND HAS ENGINES FACING BOTH FORWARD AND BACKWARD.

COORDINATE SYSTEM IN WHICH EARTH IS AT REST



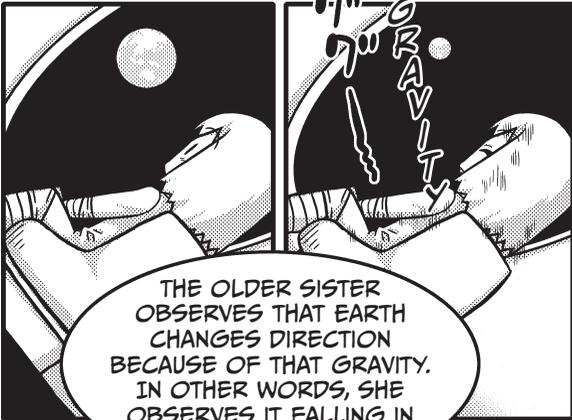
WHEN THE ROCKET TURNS AROUND TO RETURN TO EARTH...



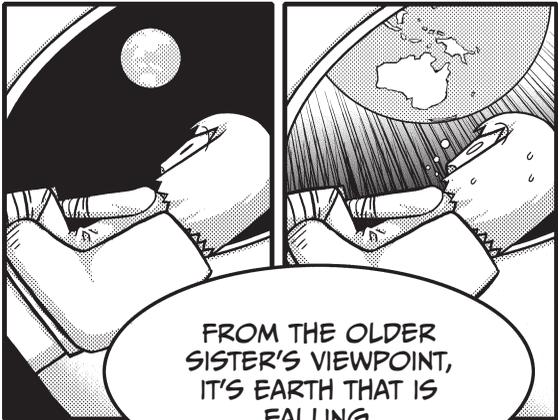
THE OLDER SISTER INSIDE FEELS AS IF SHE IS BEING SUBJECTED TO STRONG GRAVITY RATHER THAN THINKING THAT SHE IS BEING DECELERATED AND ACCELERATED.

BUT THE ROCKET ACTUALLY DECELERATES AND ACCELERATES TO TURN AROUND.

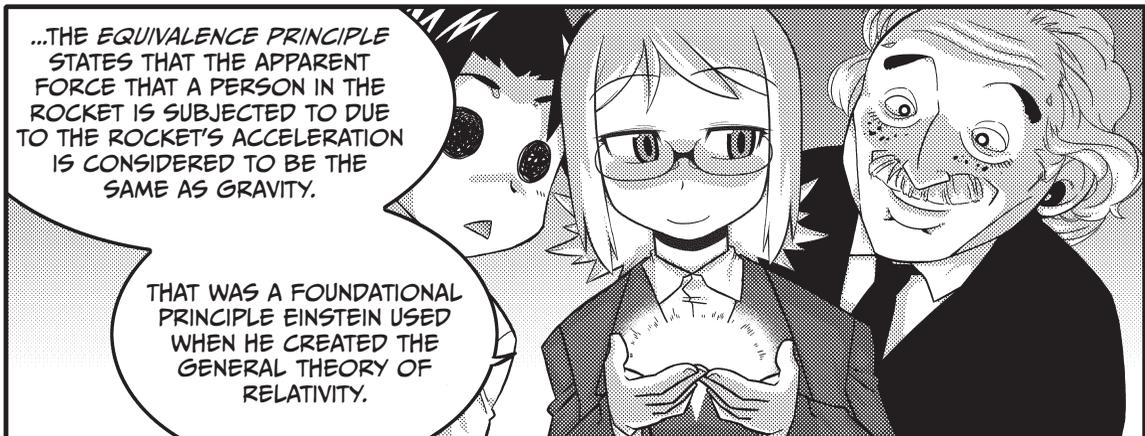
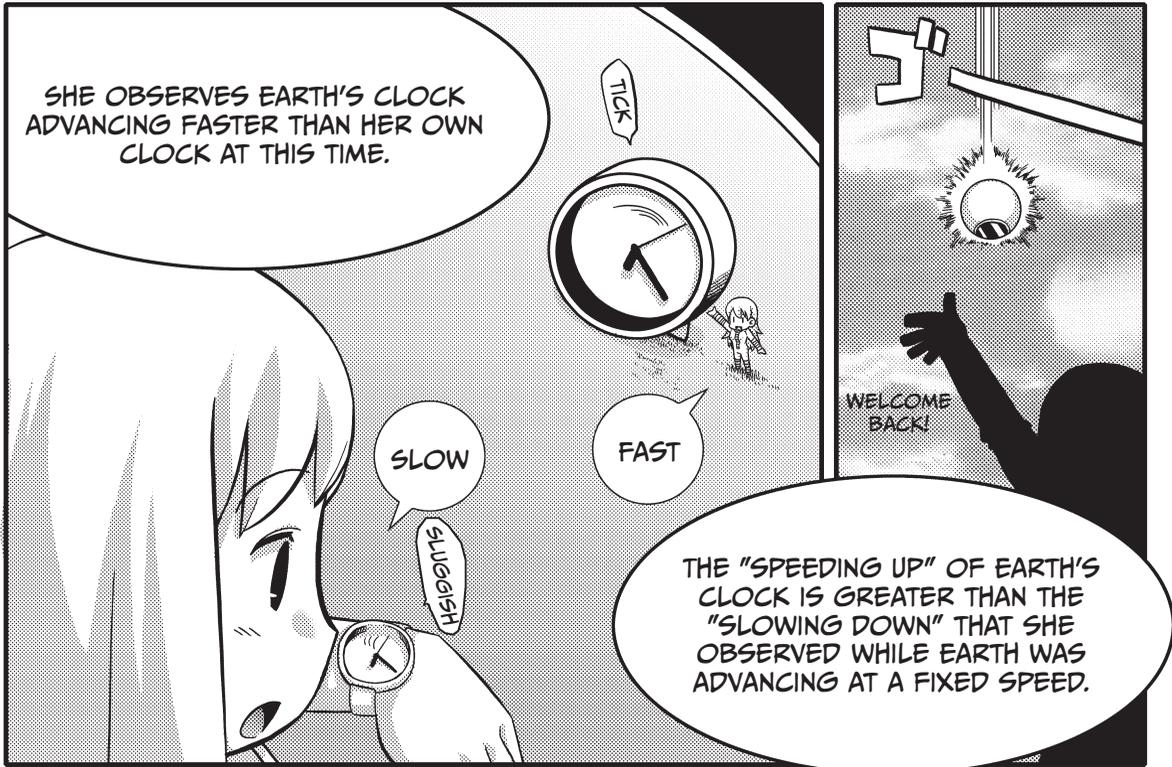
GRAVITY

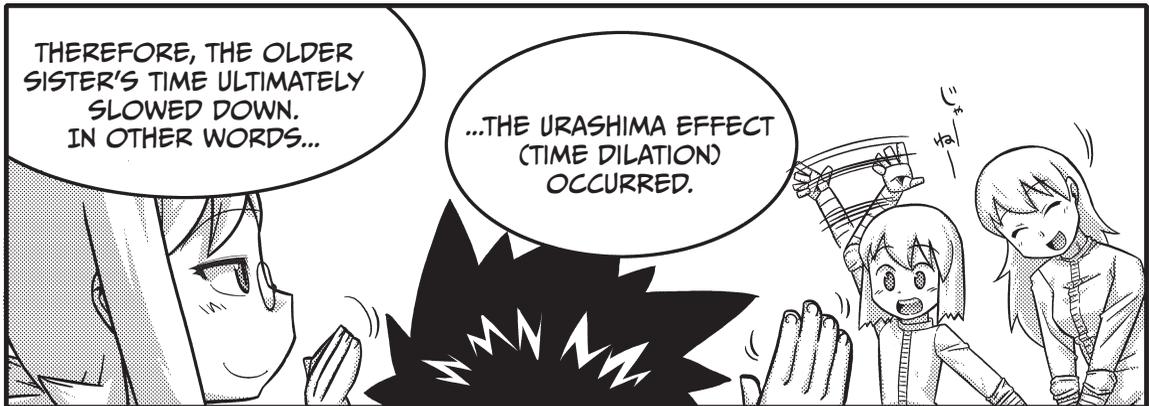


THE OLDER SISTER OBSERVES THAT EARTH CHANGES DIRECTION BECAUSE OF THAT GRAVITY. IN OTHER WORDS, SHE OBSERVES IT FALLING IN HER DIRECTION.

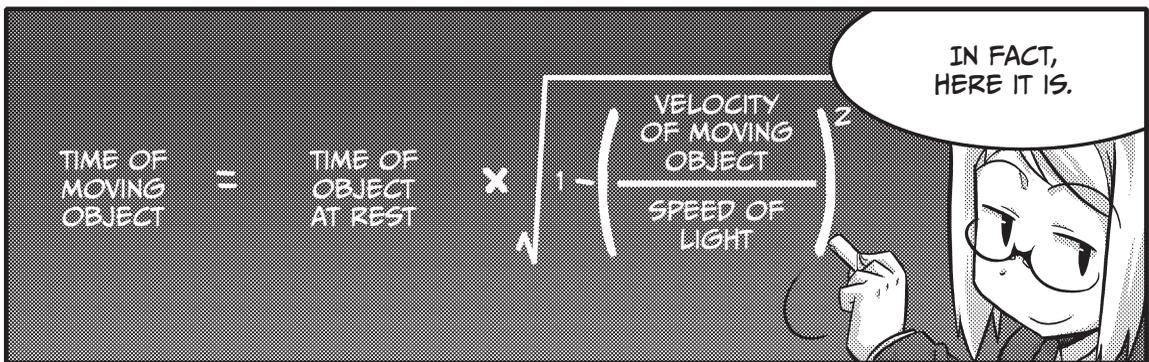
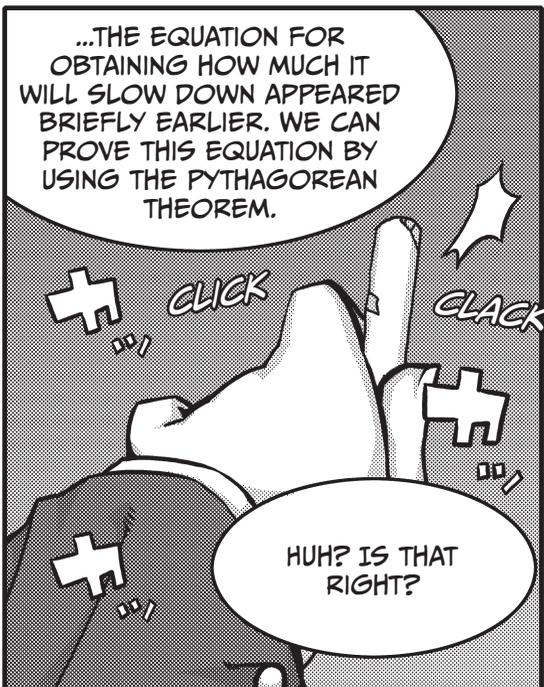
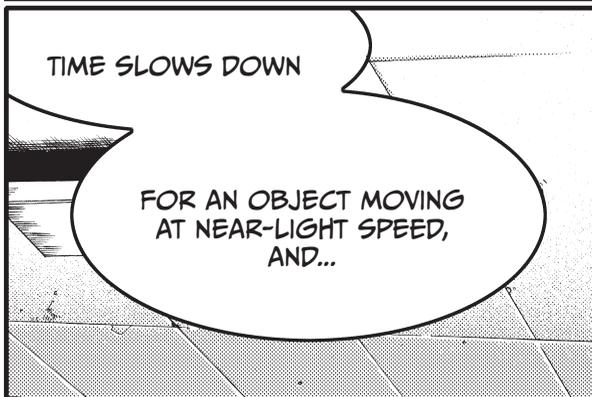


FROM THE OLDER SISTER'S VIEWPOINT, IT'S EARTH THAT IS FALLING.





4. LOOKING AT THE SLOWING OF TIME USING AN EQUATION



SOMEHOW, IT'S A SURPRISINGLY SIMPLE EQUATION, ISN'T IT?

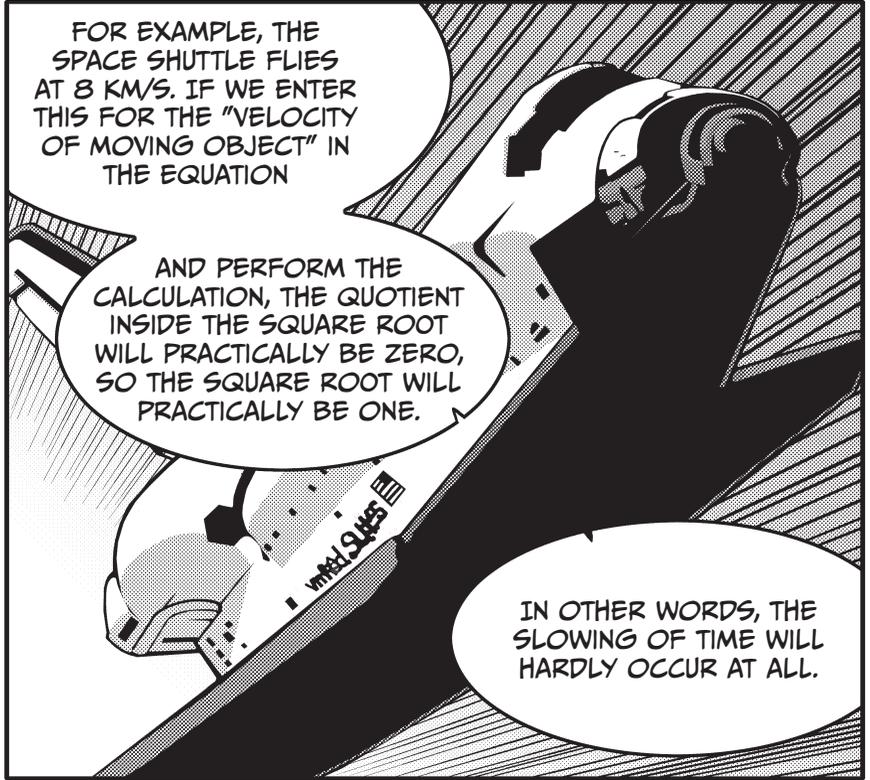


YES, IT IS.

FOR EXAMPLE, THE SPACE SHUTTLE FLIES AT 8 KM/S. IF WE ENTER THIS FOR THE "VELOCITY OF MOVING OBJECT" IN THE EQUATION

AND PERFORM THE CALCULATION, THE QUOTIENT INSIDE THE SQUARE ROOT WILL PRACTICALLY BE ZERO, SO THE SQUARE ROOT WILL PRACTICALLY BE ONE.

IN OTHER WORDS, THE SLOWING OF TIME WILL HARDLY OCCUR AT ALL.



BUT IF WE DO THE CALCULATION USING 90% OF THE SPEED OF LIGHT, OR 270,000 KM/S, THE SQUARE ROOT ITSELF...

HERE, YOU DO IT MINAGI.

$\frac{t_0}{T}$

WHAT...  
UM...ER...



$$\sqrt{1 - \left(\frac{270000}{300000}\right)^2}$$

THEREFORE...

= 0.43588989

GOOD, GOOD.

YOU UNDERSTAND IT PRETTY WELL, DON'T YOU?

THEY'RE NOT ALL SIGNIFICANT DIGITS... BUT YOU DID FINE.



ONE YEAR IS  
60 SECONDS x  
60 MINUTES x 24  
HOURS x 365 DAYS  
= 31,536,000  
SECONDS...

AND CALCULATING  
31,536,000  
x 0.4358898943, WE  
FIND THAT THE TIME OF  
THE OBJECT AT REST IS  
13,746,223 SECONDS.

IF WE CONVERT  
THIS TO DAYS...

...IT'S APPROXIMATELY  
159 DAYS.

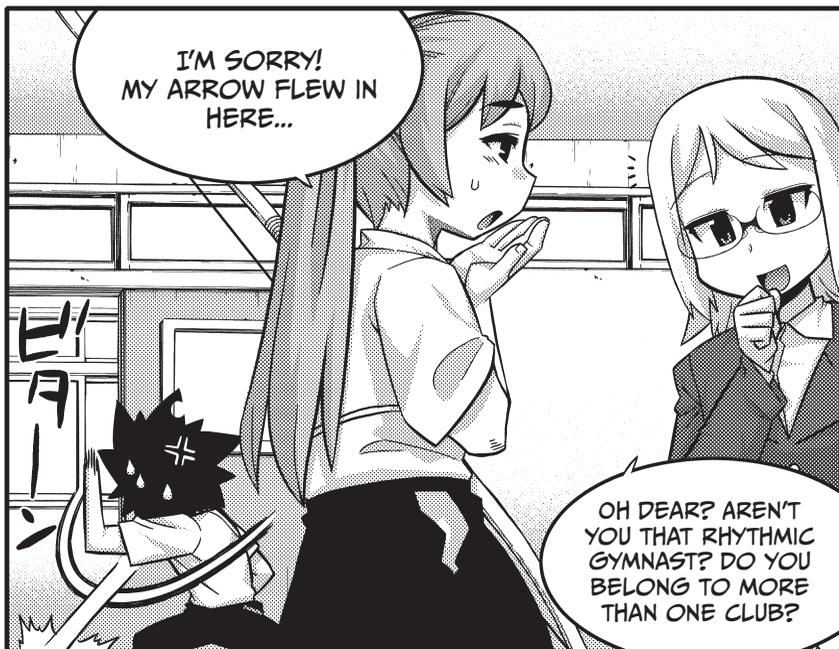
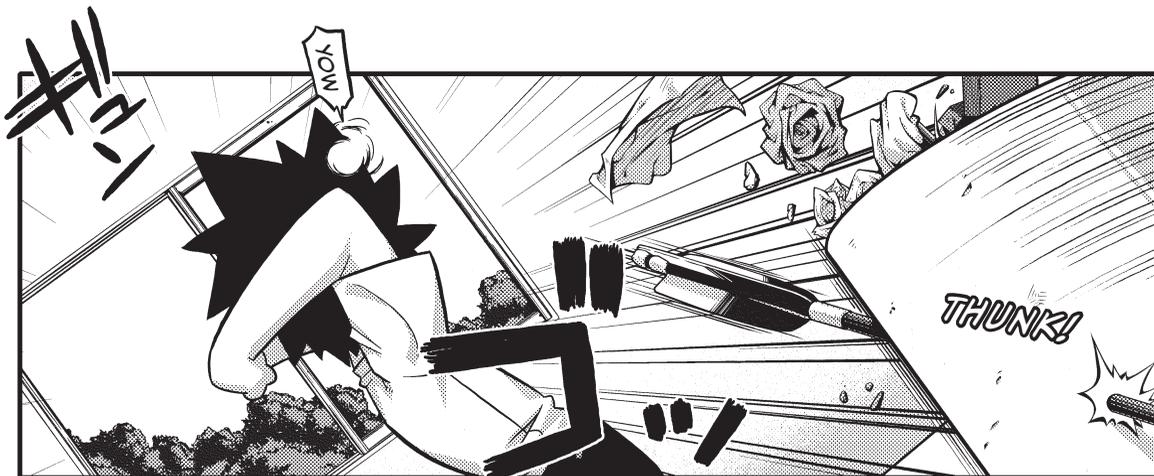
79  
79 79

PROPORTIONALLY, IT'S  
MORE THAN 2.29 TIMES  
SLOWER.

IS A YEAR FOR A  
PERSON AT REST LESS  
THAN HALF AS MUCH  
TIME FOR A PERSON  
MOVING AT NEAR-LIGHT  
SPEED?

AH.

OH?



\* FAILURE





IF THAT'S SO...  
THAT'S GREAT.  
HERE, TAKE THIS!

POKE

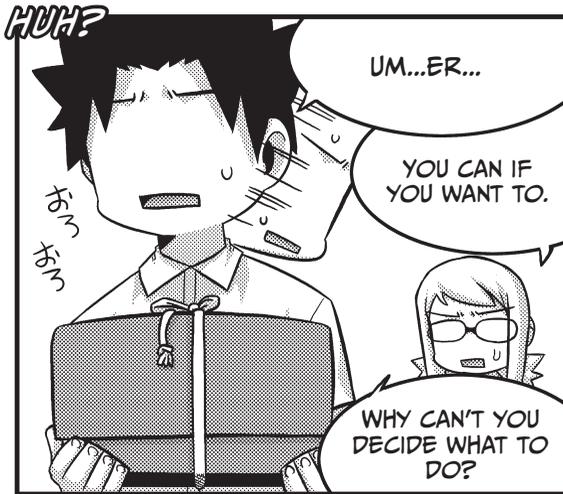


IT'S SOMETHING FOR  
YOU, MINAGI, SINCE  
YOU'RE WORKING  
SO HARD.

BUT...

...YOU MUST  
NEVER OPEN  
IT!

OKAY?



HUHP

UM...ER...

YOU CAN IF  
YOU WANT TO.

WHY CAN'T YOU  
DECIDE WHAT TO  
DO?



WELL, I SHOULD BE  
HAPPY TO TAKE THE  
PRESENT, BUT WHEN  
SHE SAID NOT TO  
OPEN IT...

I THOUGHT SHE  
WAS KIDDING.

IT APPEARS TO BE A  
SMALL JEWEL BOX.

SHAKE  
SHAKE

STOP!



YOU WILL  
REGRET IT!



I'LL JUST  
PEEK...

LISTEN TO THE  
PERSON WITH  
EXPERIENCE!

## USING THE PYTHAGOREAN THEOREM TO PROVE TIME DILATION

We learned that according to the theory of relativity, time slows down for an object that is moving at a velocity close to the speed of light. But how much does time slow? When we used the Pythagorean theorem earlier, we considered this question using a triangle. Now we can consider it using a formula.

Let  $t$  denote the amount of time that has passed according to Space-Station Man looking at the rocket's light clock and  $t'$  denote the amount of time that has passed according to Rocket Man looking at his own clock (see Figure 2-1).

$t$ : Time observed by Space-Station Man

$t'$ : Time observed by Rocket Man

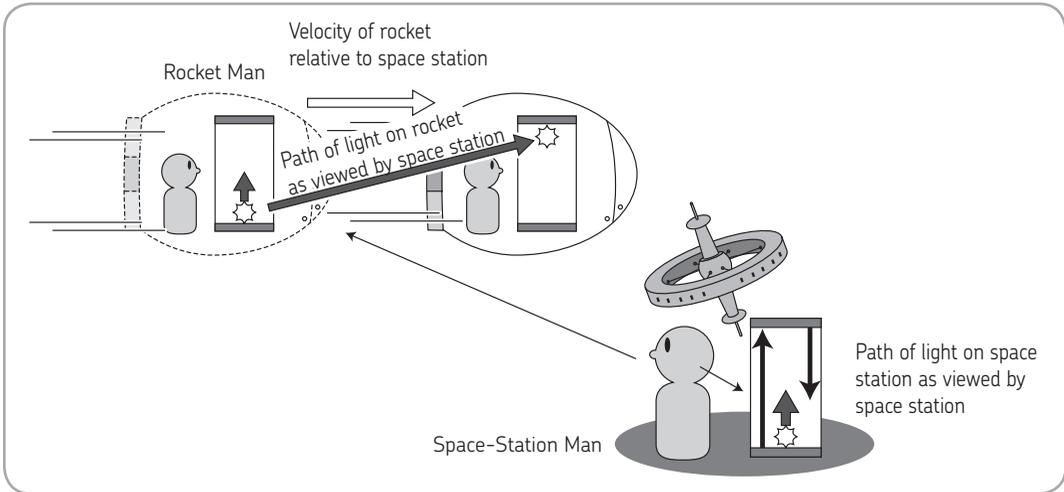


Figure 2-1: Rocket Man and Space-Station Man

When Rocket Man observes his own light clock, the light just goes up and down because the light clock is moving together with Rocket Man. Therefore, if  $c$  denotes the speed of light, when the light advances by the height of the light clock, it will have moved a distance of  $ct'$ .

Now if Space-Station Man observes the movement of the light in the rocket's light clock, the light, of course, moves at the speed of light  $c$  along an upward slanted path accompanying the movement of the rocket. That slanted line points towards the mirror (at the top) of the rocket's light clock. Measured using Space-Station Man's time  $t$ , that distance is  $ct$ . Similarly, since Space-Station Man sees the bottom of the rocket's light clock (from where the light was emitted) moving horizontally at the rocket's velocity  $v$ , the bottom will move by a distance of  $vt$  to the right in the time  $t$  that the light takes to reach the top.

This determines the three sides of a triangle (see Figure 2-2).

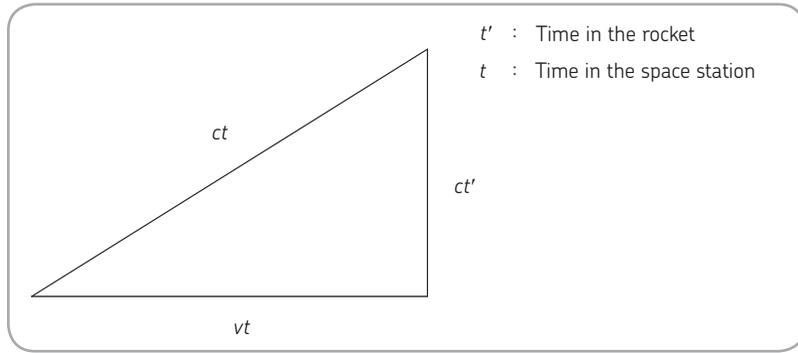


Figure 2-2: Distances moved by light as viewed on rocket and space station, expressed as sides of a right triangle

Therefore, from the Pythagorean theorem, we have  $c^2t^2 = c^2t'^2 + v^2t^2$ .  
 Move the  $v^2$  term to the left side of the equation:

$$(c^2 - v^2)t^2 = c^2t'^2$$

And switch the left and right sides:

$$c^2t'^2 = (c^2 - v^2)t^2$$

Dividing by  $c^2$ , we now have this:

$$t'^2 = \left(1 - \frac{v^2}{c^2}\right)t^2$$

Now take the square root of both sides and use the positive solution:

$$t' = \sqrt{1 - \frac{v^2}{c^2}} \times t$$

This is the relationship between Rocket Man's time  $t'$  and Space-Station Man's time  $t$ .

Note that  $t' < t$  since  $\sqrt{1 - \frac{v^2}{c^2}} < 1$ .

A second (1 s) measured on the Rocket Man's light clock thus corresponds to a longer time measured by the Space-Station Man. So Space-Station Man sees the Rocket Man's clock tick off seconds at a slower rate than his own. In other words, time advances more slowly for Rocket Man than for Space-Station Man.

It is also apparent by considering the term

$$\sqrt{1 - \frac{v^2}{c^2}}$$

that this time-slowing effect is greater the closer  $v$  is to  $c$ .

With this formula, we can calculate the time dilation effect for an object moving at any relative speed.

## HOW MUCH DOES TIME SLOW DOWN?

We have learned that time slows down for a moving object. Let's now determine exactly how much it slows down. We will use space travel as an example for this calculation.

As we saw above, the slowing of time is related to the velocity of the moving object. Recall that the closer the velocity of the moving object is to the speed of light, the greater the time-slowing effect becomes.

As a destination for our space travel, let's pick the star that is closest to our Sun—Alpha Centauri (see Figure 2-3). If we chose a destination inside our solar system, we wouldn't put the theory of relativity to the test because we can go to Mars or Venus in just a few years even with current technology, going nowhere near the speed of light.

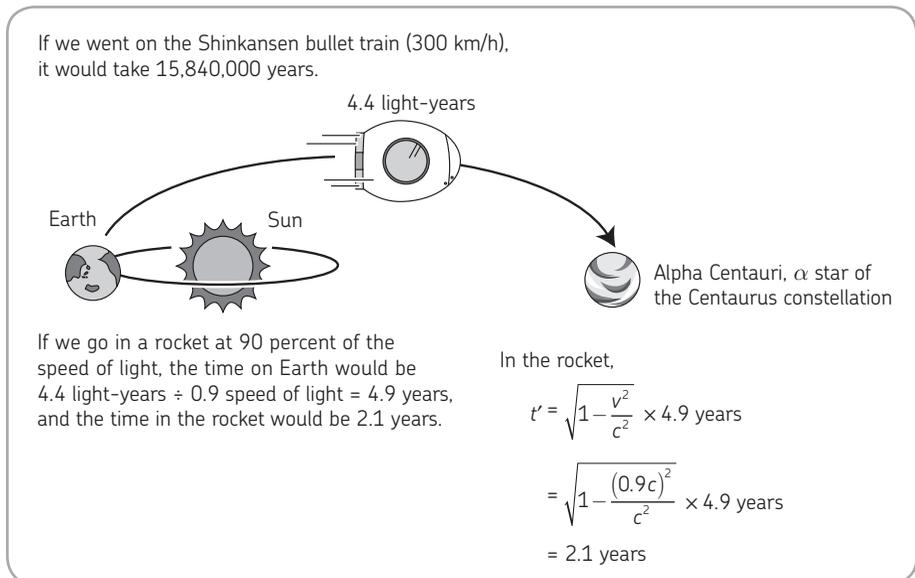


Figure 2-3: Traveling to Alpha Centauri

Alpha Centauri (the  $\alpha$  star of the Centaurus constellation) is 4.4 light-years away from Earth. A light-year, which is the distance that light travels in one year, is approximately 9,460,800,000,000 km.\* Traveling 4.4 light-years on the Shinkansen bullet train (at 300 km/h) would take approximately 15,840,000 years. But if we fly that distance to Alpha Centauri at 90 percent of the speed of light, the journey will take us only 2.1 years. This is despite the fact that 4.9 years will pass on Earth!

If astronauts were sent out from Earth to Alpha Centauri, even if they returned as soon as they got there, news reports on Earth would follow them for approximately 10 years, but when their families welcomed them home, they would have aged only 4.2 years.

This relativistic time slowing effect is even greater if you move faster.

To understand this relationship, let's consider traveling from the Milky Way galaxy, which contains our Sun, to the Andromeda galaxy (M31), which is near the Milky Way. The Andromeda galaxy is visible in a dark, clear winter sky as a faint smudge in the Andromeda constellation. It is approximately 2,500,000 light-years away. Because light from the galaxy takes 2,500,000 years to reach us, the Andromeda galaxy that we see now is actually the galaxy 2,500,000 years ago. Even if an explosion occurred in the Andromeda Galaxy today, we wouldn't see it until 2,500,000 years later; the light would still take that long to reach us.

If astronauts travel to the Andromeda galaxy at 99.99999999 percent of the speed of light, 11.2 years will pass for the one-way trip in the spaceship, but nearly 2,500,000 years will pass on Earth! Therefore, when the spaceship returns, although the astronauts will have aged 22.4 years, the people on Earth who will greet them will be from a time 5,000,000 years later.

Let's do the math: If a spaceship traveled to the Andromeda galaxy at 99.99999999 percent of the speed of light, people on Earth will have to wait 2,500,000 years for the spaceship to reach Andromeda and another 2,500,000 years for the spaceship to return. In all, 5 million years will pass before the spaceship returns to Earth. The amount of time that passes for the astronauts on the spaceship during this journey  $t'$  can be calculated using the time dilation formula that we derived above, using the time that it takes for the spaceship to travel to Andromeda as measured on earth ( $t = 2,500,000$  years) and the velocity of the spaceship ( $v = 0.9999999999c$ ).

$$t' = \sqrt{1 - \frac{v^2}{c^2}} \times t$$

$$t' = \sqrt{1 - \left(\frac{0.9999999999c}{c}\right)^2} \times t$$

$$t' = 11.2 \text{ years}$$

---

\* Let's do the math:  $\frac{300,000 \text{ km}}{1 \text{ second}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times \frac{24 \text{ hours}}{1 \text{ day}} \times \frac{365 \text{ days}}{1 \text{ year}}$   
 = 9,460,800,000 km!

For the astronauts on the spaceship, the round-trip to Andromeda will take 22.4 years, as shown in Figure 2-4. For the people on Earth, the astronauts will return only 22.4 years older than when they left 5 million years earlier!

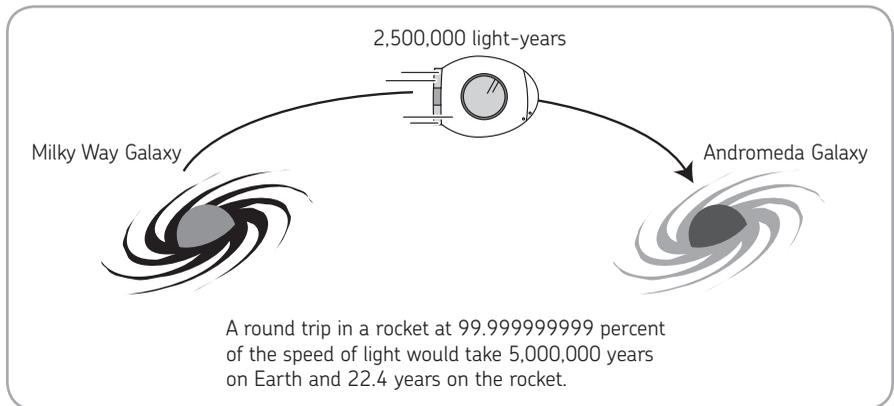
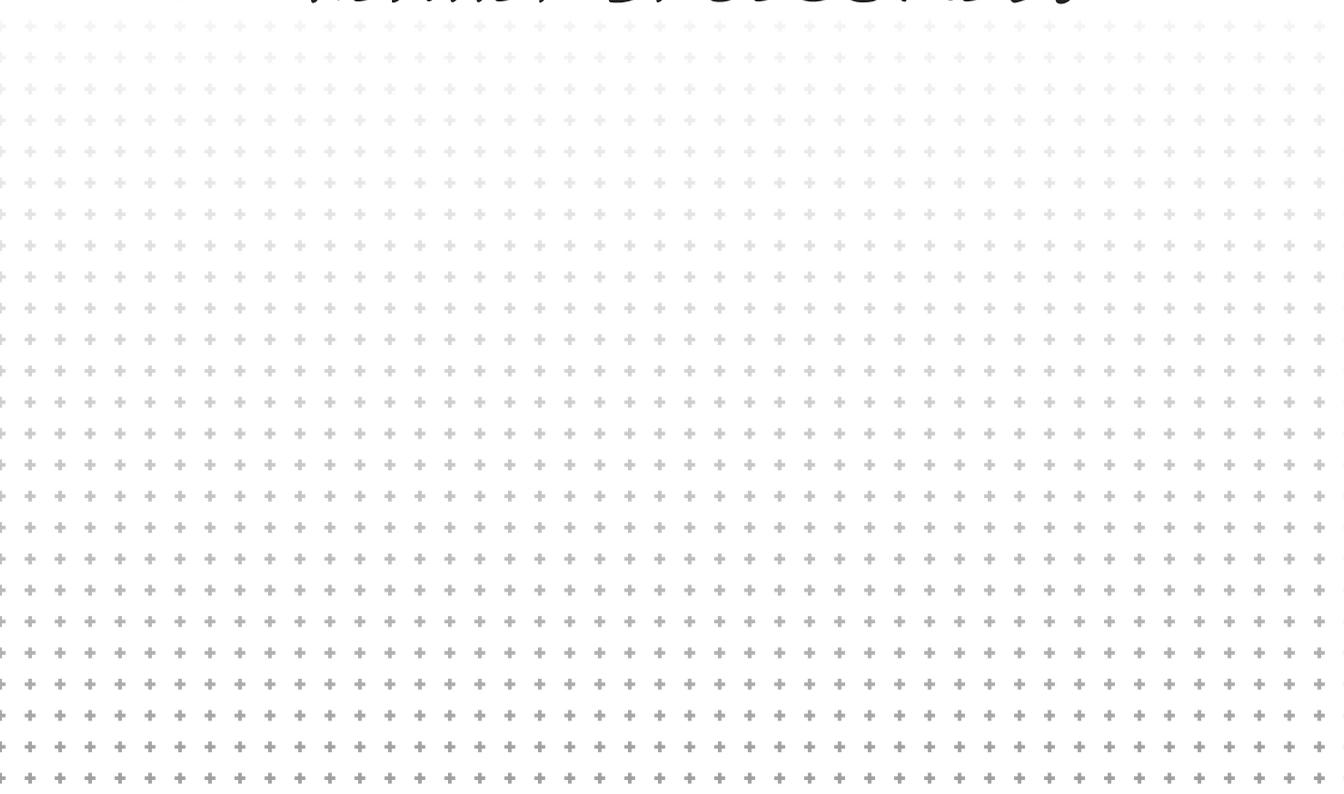


Figure 2-4: Traveling to the Andromeda galaxy



THE FASTER AN OBJECT  
MOVES, THE SHORTER AND  
HEAVIER IT BECOMES?





LOOK NOW.

CAN'T YOU FAN ME MORE STEADILY?



WHEN DID I BECOME THE EXPERT?



WELL...

WHY ARE WE AT THE POOL? WHAT'S GOING ON?

SINCE YOU OPENED THE PRESENT THAT GIRL BROUGHT YOU, WE CAN'T USE THE PHYSICS LAB FOR A WHILE.



I OPENED THAT BOX?! WHAT HAPPENED...?

SHOULD WE CALL FOR HELP?

YIKES... WHAT IS THIS?

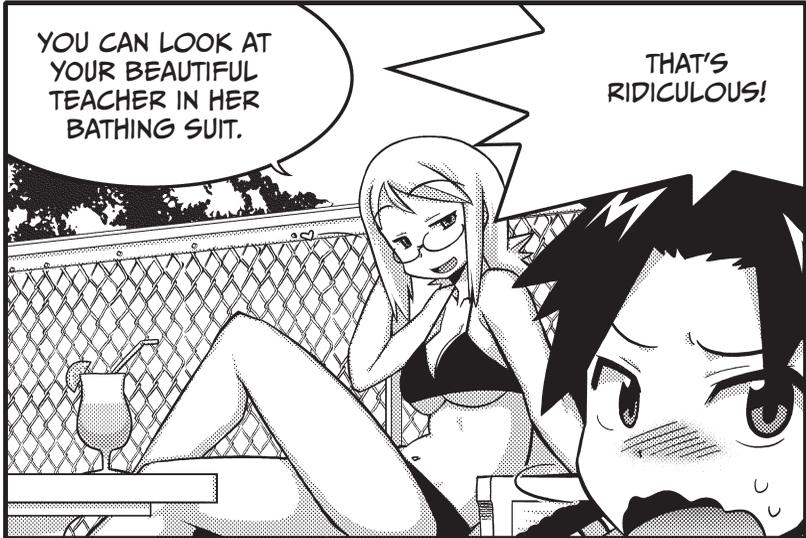
IT'S BETTER THAT YOU DON'T KNOW!



THEREFORE, THE POOL WILL BE OUR CLASSROOM TODAY.

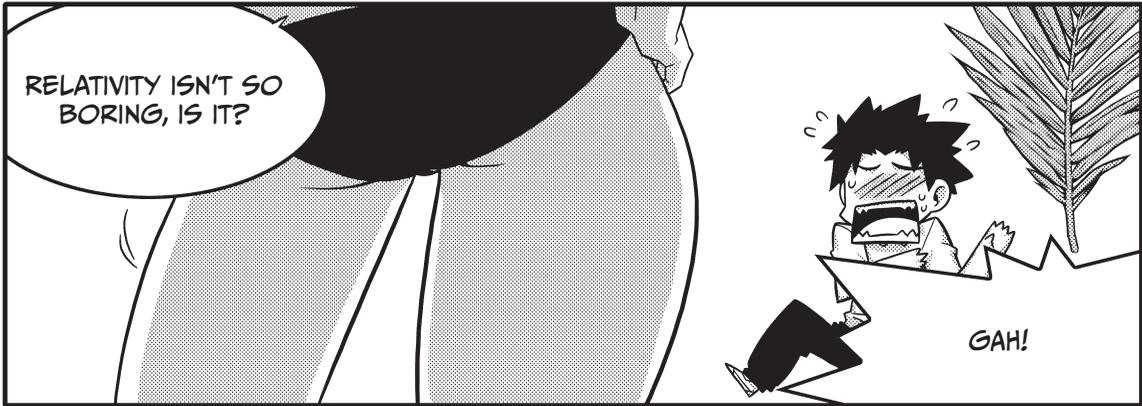
YOU'RE ACTUALLY PROBABLY PRETTY HAPPY ABOUT THAT, AREN'T YOU?

AM I?



YOU CAN LOOK AT YOUR BEAUTIFUL TEACHER IN HER BATHING SUIT.

THAT'S RIDICULOUS!



RELATIVITY ISN'T SO BORING, IS IT?



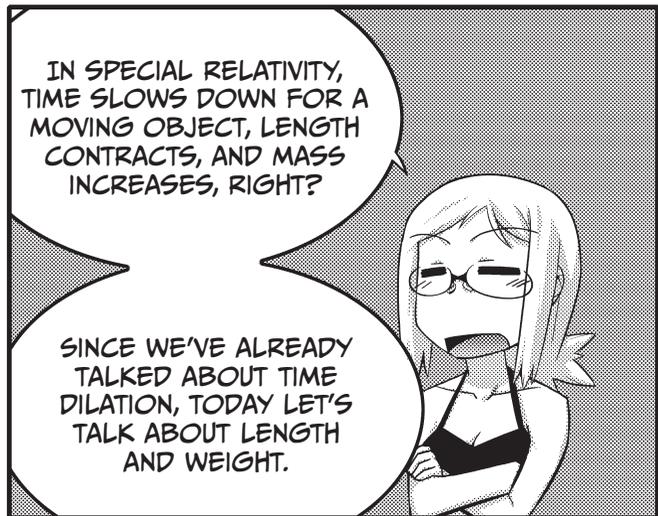
GAH!



GET ON WITH THE LESSON ALREADY, OKAY?

ALL RIGHT, ALL RIGHT.

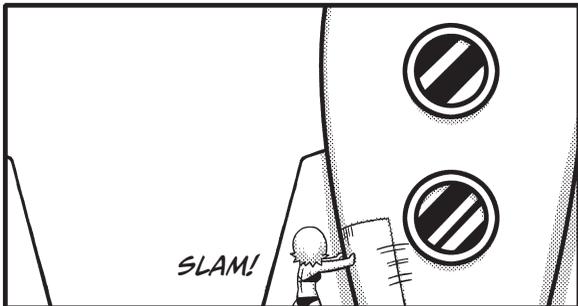
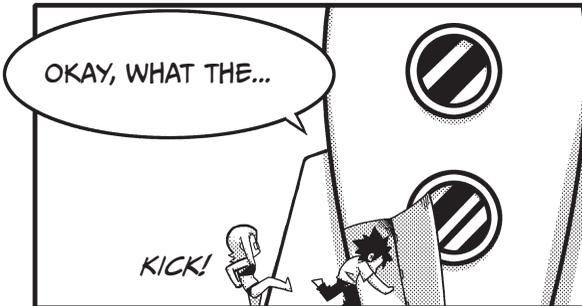
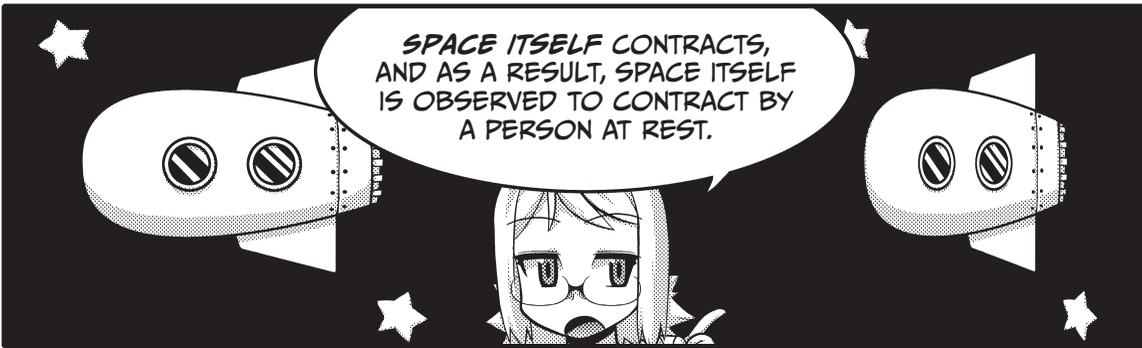
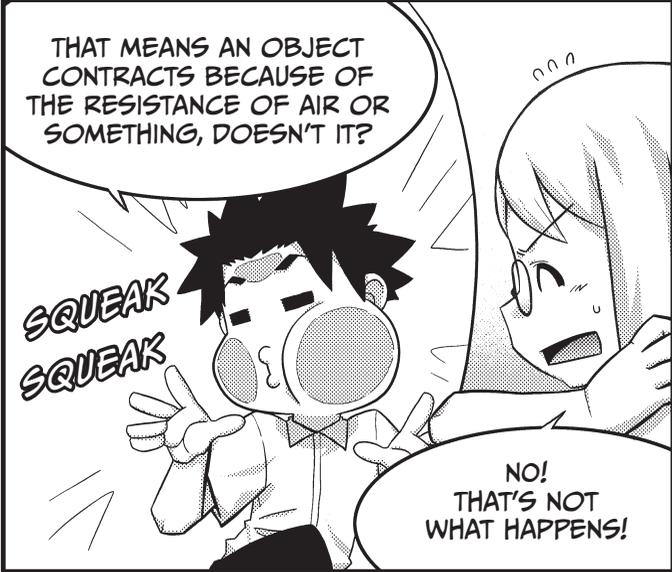
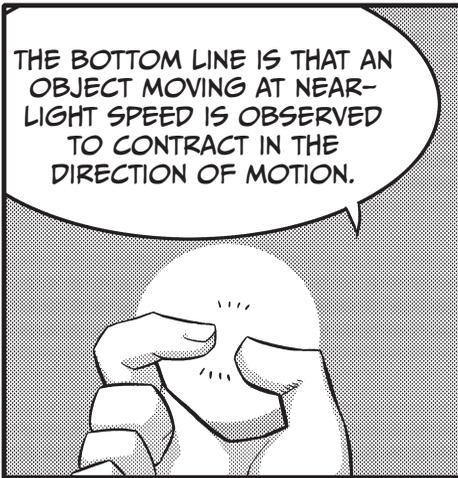
CLINK

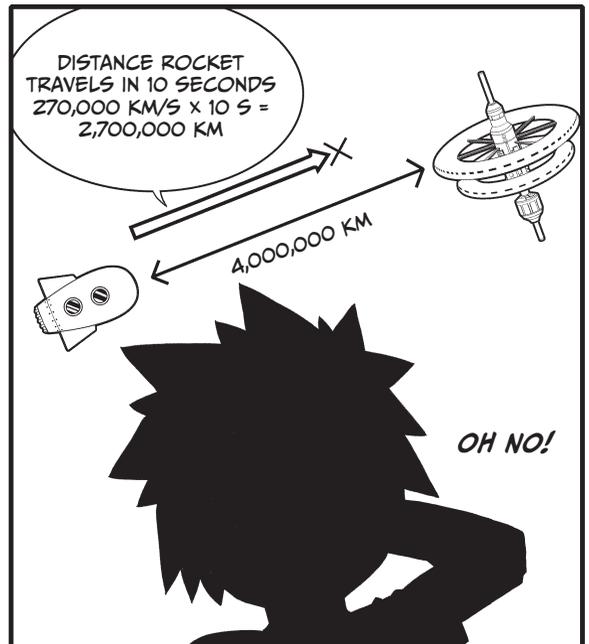
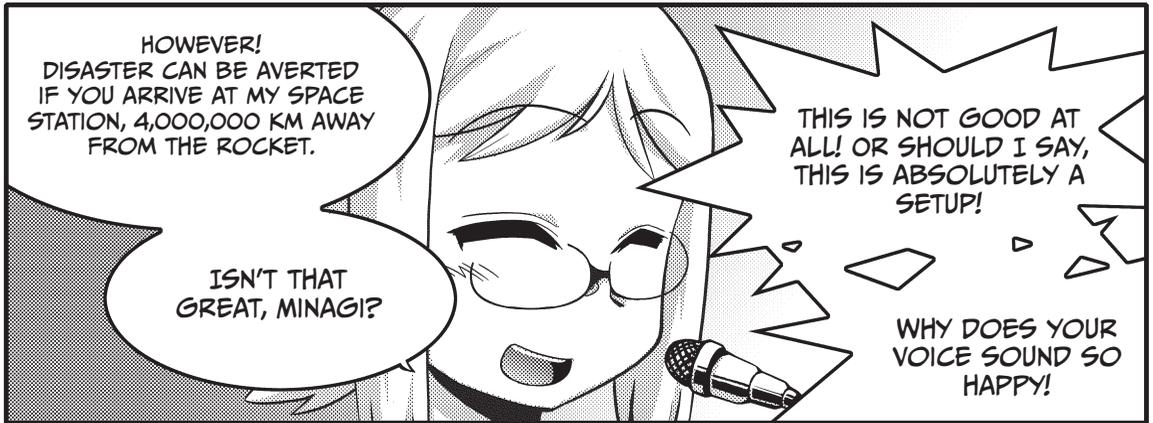


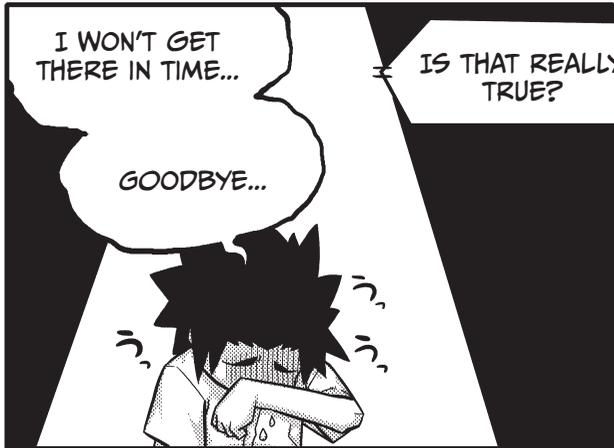
IN SPECIAL RELATIVITY, TIME SLOWS DOWN FOR A MOVING OBJECT, LENGTH CONTRACTS, AND MASS INCREASES, RIGHT?

SINCE WE'VE ALREADY TALKED ABOUT TIME DILATION, TODAY LET'S TALK ABOUT LENGTH AND WEIGHT.

1. DOES LENGTH CONTRACT WHEN YOU GO FASTER?





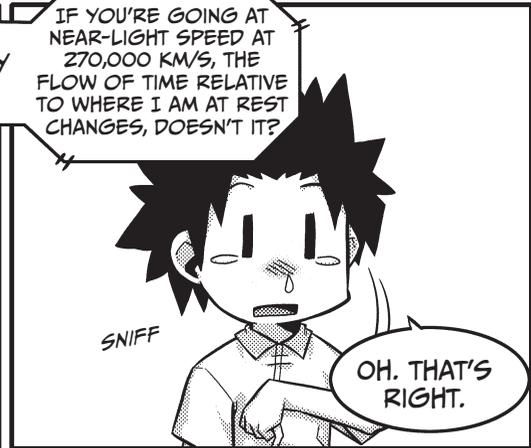


I WON'T GET THERE IN TIME...

GOODBYE...

IS THAT REALLY TRUE?

IF YOU'RE GOING AT NEAR-LIGHT SPEED AT 270,000 KM/S, THE FLOW OF TIME RELATIVE TO WHERE I AM AT REST CHANGES, DOESN'T IT?



SNIFF

OH. THAT'S RIGHT.

IF YOU CONSIDER APPLYING THIS VELOCITY IN THE "EQUATION FOR CALCULATING THE SLOWING OF TIME" THAT WE TALKED ABOUT EARLIER...

$$\text{TIME OF MOVING OBJECT} = \text{TIME OF OBJECT AT REST} \times \sqrt{1 - \left(\frac{\text{VELOCITY OF MOVING OBJECT}}{\text{SPEED OF LIGHT}}\right)^2}$$

THE ROCKET'S TIME FLOWS AT APPROXIMATELY 0.44 TIMES THAT OF THE SPACE STATION, WHICH IS AT REST.

OF COURSE...

WHEN OBSERVED BY ME IN THE SPACE STATION,

IT WILL TAKE  $4,000,000 \text{ KM} \div 270,000 \text{ KM/S}$  OR APPROXIMATELY 14.8 SECONDS FOR THE ROCKET TO ARRIVE.

MINAGI RIDING IN THE ROCKET FLYING AT 270,000 KM/S

MISS URAGA IN THE SPACE STATION

DISTANCE BETWEEN SPACE STATION AND ROCKET IS 4,000,000 KM

HOWEVER, FOR YOU, MINAGI, IN THE ROCKET, IT WILL TAKE  $14.8 \text{ SECONDS} \times 0.44$  OR APPROXIMATELY 6.5 SECONDS TO REACH THE SPACE STATION.

$4,000,000 \text{ KM} \div 270,000 \text{ KM/S} = 14.8 \text{ S}$

OH GOOD, I GOT THERE IN TIME...

BUT I'M NOT REALLY HAPPY WITH THAT EXPLANATION.



THAT WAS JUST THE STORY OF TIME INSIDE THE ROCKET, RIGHT?

I MADE IT! I GOT HERE IN TIME!

IF I ARRIVED WITHIN 10 SECONDS TO A LOCATION 4,000,000 KM AWAY, EVEN THOUGH I WAS TRAVELING AT 270,000 KM/S...

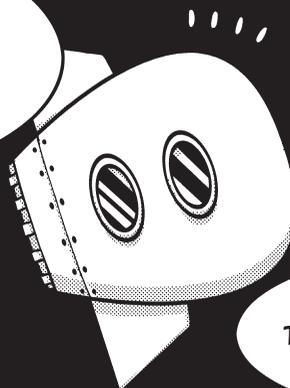
TOO LATE.

KABOOM

HMMM...THAT MEANS THAT LENGTH CONTRACTS.

WHAT THE...?!

FIRST, WHEN THE ROCKET THAT IS FLYING AT 270,000 KM/S IS OBSERVED BY SOMEONE AT REST OUTSIDE OF IT, THE ROCKET'S LENGTH IS OBSERVED TO BE CONTRACTED.



SCRUNCH

THAT'S WHAT YOU SAID.

THE CONTRACTION OF LENGTH IS OBTAINED BY USING THIS EQUATION.

$$\text{LENGTH WHEN MOVING} = \text{LENGTH WHEN AT REST} \times \sqrt{1 - \left(\frac{\text{MOVING VELOCITY}}{\text{SPEED OF LIGHT}}\right)^2}$$

THIS EQUATION LOOKS LIKE THE EQUATION FOR TIME DILATION.

BECAUSE LENGTH IS PROPORTIONAL TO VELOCITY,  $L = v \times t$ . THE PROPORTION THAT LENGTH CONTRACTS MUST BE THE SAME THAT TIME DILATES IN ORDER FOR VELOCITY TO REMAIN THE SAME.

RIGHT...THAT'S THE IMPORTANT POINT.

MEANING?

WHEN VIEWED FROM THE ROCKET,

THE SURROUNDING SPACE CAN BE THOUGHT OF AS MOVING AND THE SPACE STATION AS COMING CLOSER.

ガラガラ

THAT'S BECAUSE ACCORDING TO RELATIVITY, THE ROCKET CAN BE CONSIDERED TO BE AT REST, RIGHT?

THE SPACE SURROUNDING THE ROCKET CONTRACTED!

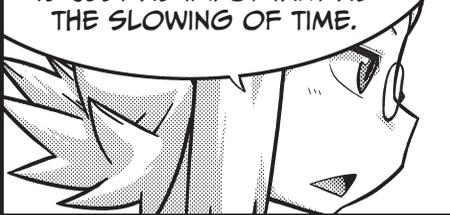


THE DISTANCE OF 4,000,000 KM TO THE SPACE STATION CONTRACTS TO APPROXIMATELY 1,740,000 KM ACCORDING TO THE PREVIOUS EQUATION.



THEREFORE, YOU CAN GO THERE IN  $1,740,000 \text{ KM} \div 270,000 \text{ KM/S}$  OR APPROXIMATELY 6.5 SECONDS.

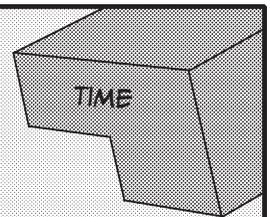
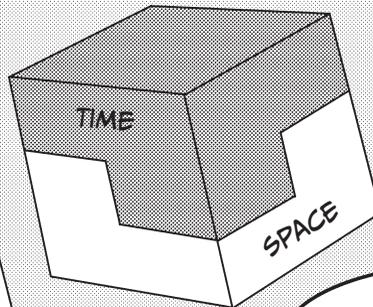
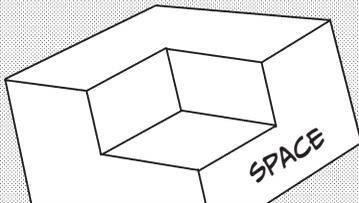
IN THE THEORY OF RELATIVITY, THE CONTRACTION OF LENGTH IS JUST AS IMPORTANT AS THE SLOWING OF TIME.



I SEE...



AND TIME AND SPACE, WHICH NORMALLY ARE CONSIDERED SEPARATELY, MUST BE CONSIDERED TOGETHER.



THIS IS A REVOLUTIONARY PART OF RELATIVITY THEORY.

2. DO YOU GET HEAVIER WHEN YOU GO FASTER?

NEXT WE'LL TALK ABOUT GETTING HEAVIER WHEN YOU GO FASTER.

YOU ALSO GET HEAVIER? THAT DOESN'T REALLY SEEM REAL...

LOOK, AS USUAL, THIS EFFECT IS NOT NOTICEABLE IF YOU'RE NOT MOVING AT NEAR-LIGHT SPEED.

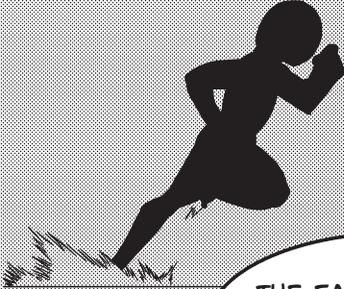
I DIDN'T MEAN ANYTHING BY THAT GLANCE! HONEST!

LIGHT MOVES AT 300,000 KM/S, AND THIS IS THE MAXIMUM VELOCITY IN THE NATURAL WORLD.

IN OTHER WORDS, IT'S THE UPPER LIMIT OF VELOCITY, WHICH CANNOT BE EXCEEDED NO MATTER HOW HARD YOU TRY.

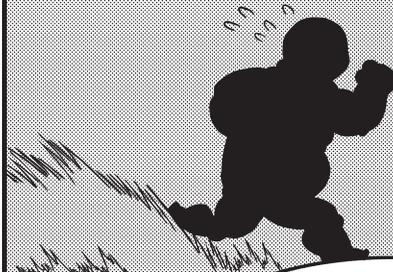
THAT'S STRANGE, ISN'T IT?

WHY CAN'T ANYTHING GO FASTER THAN THAT?



THE FASTER YOU GO,  
THE HEAVIER YOU  
BECOME.

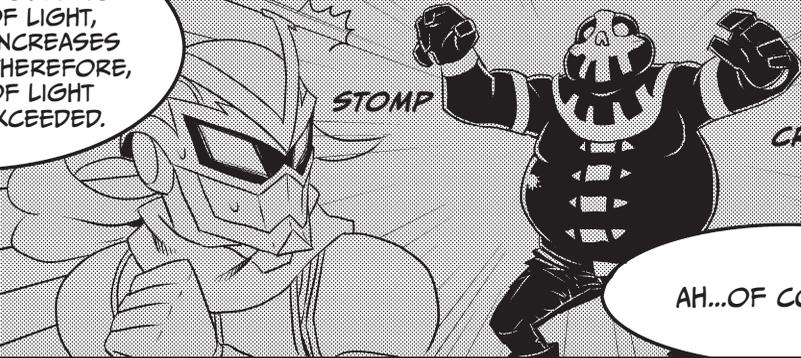
HEAVIER...WHAT HAPPENS?



YOU STEADILY GET  
HEAVIER, AND YOUR  
SPEED BECOMES  
HARDER TO INCREASE.

AS YOU GET CLOSER TO  
THE SPEED OF LIGHT,  
YOUR WEIGHT INCREASES  
WITHOUT LIMIT. THEREFORE,  
THE SPEED OF LIGHT  
CANNOT BE EXCEEDED.

WOW!



AH...OF COURSE!

質量\*

重さ\*

NOW, ALTHOUGH WE'VE  
BEEN USING THE FAMILIAR  
WORD *WEIGHT*, STRICTLY  
SPEAKING, THE *MASS*  
INCREASES.

I THOUGHT  
WEIGHT AND  
MASS WERE THE  
SAME. WHAT'S THE  
DIFFERENCE?

\* MASS

\* WEIGHT



IF WE'RE ONLY TALKING ABOUT EVERYDAY LIFE ON EARTH, WEIGHT AND MASS ARE BOTH THE SAME...

...BUT SINCE WEIGHT IS THE MAGNITUDE OF THE GRAVITATIONAL FORCE ACTING ON AN OBJECT, WEIGHT WILL DIFFER ON EARTH AND ON THE SURFACE OF THE MOON.

MINAGI ON THE SURFACE OF THE MOON WEIGHS 25 LBS.



THE MOON'S GRAVITY IS 1/6 THAT OF EARTH'S.

MINAGI ON EARTH WEIGHS 150 LBS.

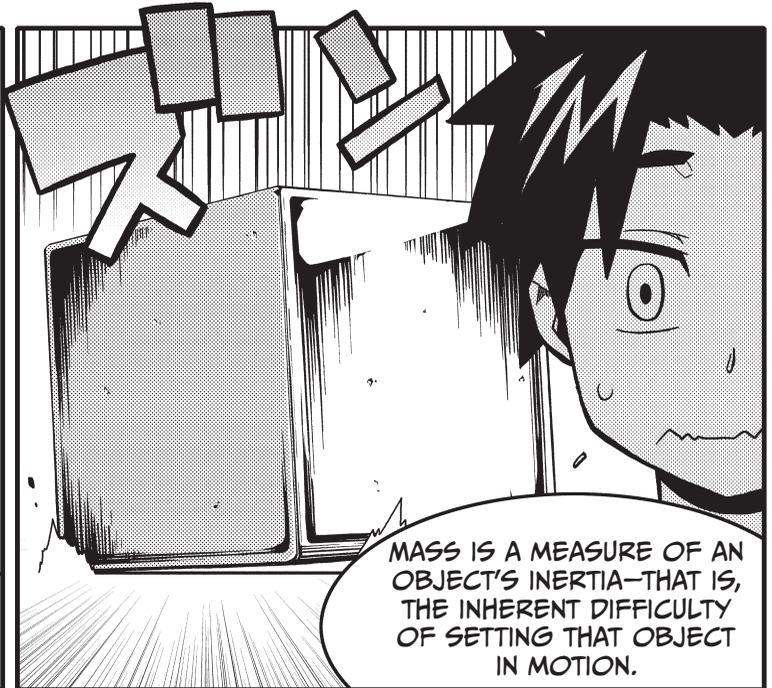


I SEE... BUT MASS?

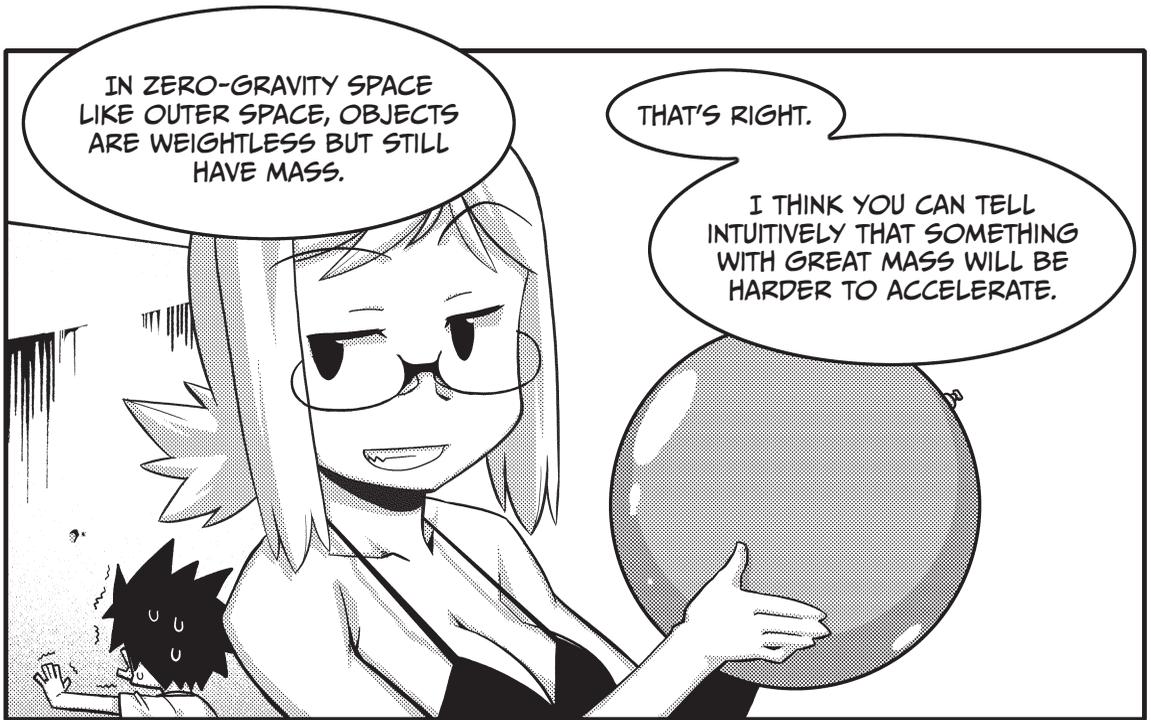
IN ZERO-GRAVITY SPACE, ALTHOUGH THE WEIGHT IS ZERO, A FORCE IS REQUIRED TO SET SOMETHING IN MOTION, RIGHT?



THAT'S RIGHT, BUT...?



MASS IS A MEASURE OF AN OBJECT'S INERTIA—THAT IS, THE INHERENT DIFFICULTY OF SETTING THAT OBJECT IN MOTION.



IN ZERO-GRAVITY SPACE LIKE OUTER SPACE, OBJECTS ARE WEIGHTLESS BUT STILL HAVE MASS.

THAT'S RIGHT.

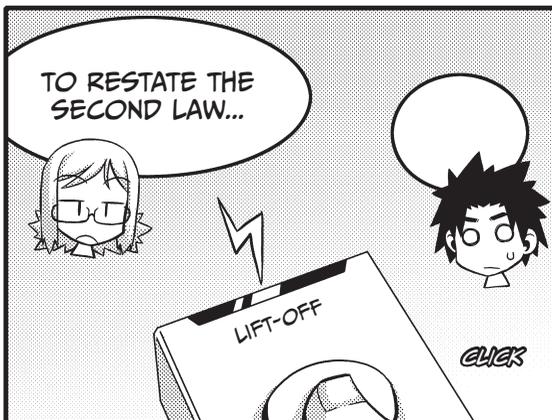
I THINK YOU CAN TELL INTUITIVELY THAT SOMETHING WITH GREAT MASS WILL BE HARDER TO ACCELERATE.

IN NEWTON'S EQUATION OF MOTION, THE MASS OF AN OBJECT IS INVERSELY PROPORTIONAL TO THE ACCELERATION, WHICH IS THE RATE AT WHICH ITS VELOCITY INCREASES.

NOW THAT YOU MENTION IT, I REMEMBER THAT ONE...

NEWTON'S THREE LAWS OF MOTION

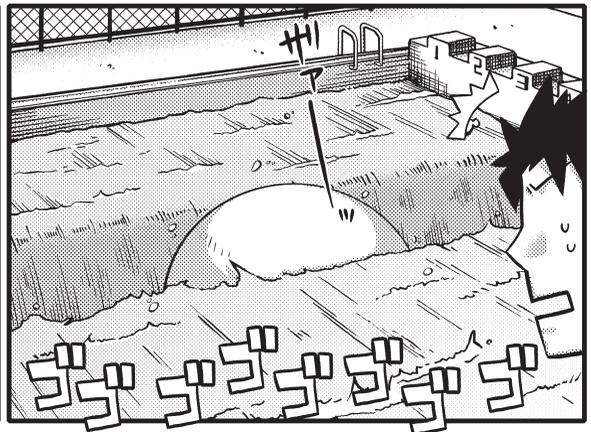
|                                     |   |
|-------------------------------------|---|
| 1st Law: Law of inertia             | An object not subjected to a force remains at rest or continues in uniform linear motion.   |
| 2nd Law: Equation of motion         | The acceleration of an object is directly proportional to an applied force and inversely proportional to its mass.                            |
| 3rd Law: Law of action and reaction | If a force is exerted on an object, the object on which that force was exerted will exert a force of the same size in the opposite direction. |



TO RESTATE THE SECOND LAW...

LIFT-OFF

CLICK



IF THE PROPULSIVE FORCE DOES NOT CHANGE FOR A ROCKET FLYING WITH AN ACCELERATION OF  $10 \text{ m/s}^2$

BUT IF ITS MASS DOUBLES, THE ACCELERATION WILL BECOME  $5 \text{ m/s}^2$ .

YOU'RE SAYING THAT WHEN THE MASS INCREASES, IT WILL BE ACCORDINGLY HARDER TO ACCELERATE.

THE CLOSER THE ROCKET GETS TO THE SPEED OF LIGHT, THE GREATER ITS MASS BECOMES...

AND IF YOU TRY TO FURTHER ACCELERATE THE ROCKET BY EXERTING ADDITIONAL FORCE...

...THE MASS WILL ALSO INCREASE FURTHER, AND IT WILL BECOME MORE AND MORE DIFFICULT TO ACCELERATE.

IF YOU CONTINUE IN THIS WAY, THE MASS WILL INCREASE WITHOUT LIMIT AS THE ROCKET'S VELOCITY GETS CLOSER TO THE SPEED OF LIGHT.

RELATIONSHIP BETWEEN VELOCITY AND MASS

MASS

VELOCITY RATIO (VELOCITY/SPEED OF LIGHT)

IF WE ASSUME FOR ARGUMENT'S SAKE THAT THE ROCKET'S VELOCITY REACHED THE SPEED OF LIGHT...

ITS MASS WOULD BECOME INFINITELY LARGE. THEREFORE, THE SPEED OF LIGHT CANNOT BE EXCEEDED.

BUT DOESN'T IT REALLY SEEM ODD THAT FOR SOMETHING THAT WOULD NORMALLY INCREASE STEADILY IN VELOCITY AS ENERGY IS CONTINUOUSLY PROVIDED TO IT...

...ENDS UP HAVING ITS MASS INCREASE RATHER THAN ITS VELOCITY?

CATCH

PSH PSH  
PSH

THAT'S RIGHT.

ACTUALLY, THERE ARE ALSO TWO IMPORTANT LAWS KNOWN AS THE LAW OF CONSERVATION OF MASS AND THE LAW OF CONSERVATION OF ENERGY.

SWEAT SWEAT

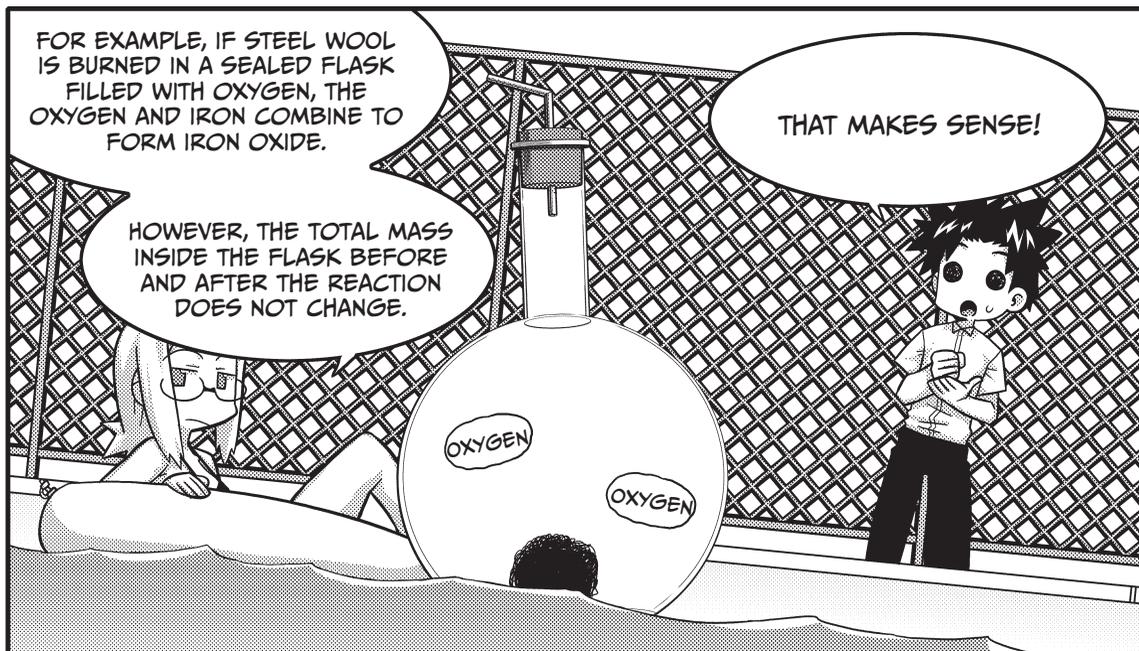
I'M SORRY...WHAT DID YOU JUST SAY?

THE LAW OF CONSERVATION OF MASS SAYS THAT THE TOTAL MASS OF SUBSTANCES WILL NOT CHANGE REGARDLESS OF ANY CHEMICAL REACTIONS THAT OCCUR AMONG THEM.

SPLASH

THERE'S ABSOLUTELY NOTHING MORE I CAN DO WITH THAT INNER TUBE.

MMMMHMMM...



FOR EXAMPLE, IF STEEL WOOL IS BURNED IN A SEALED FLASK FILLED WITH OXYGEN, THE OXYGEN AND IRON COMBINE TO FORM IRON OXIDE.

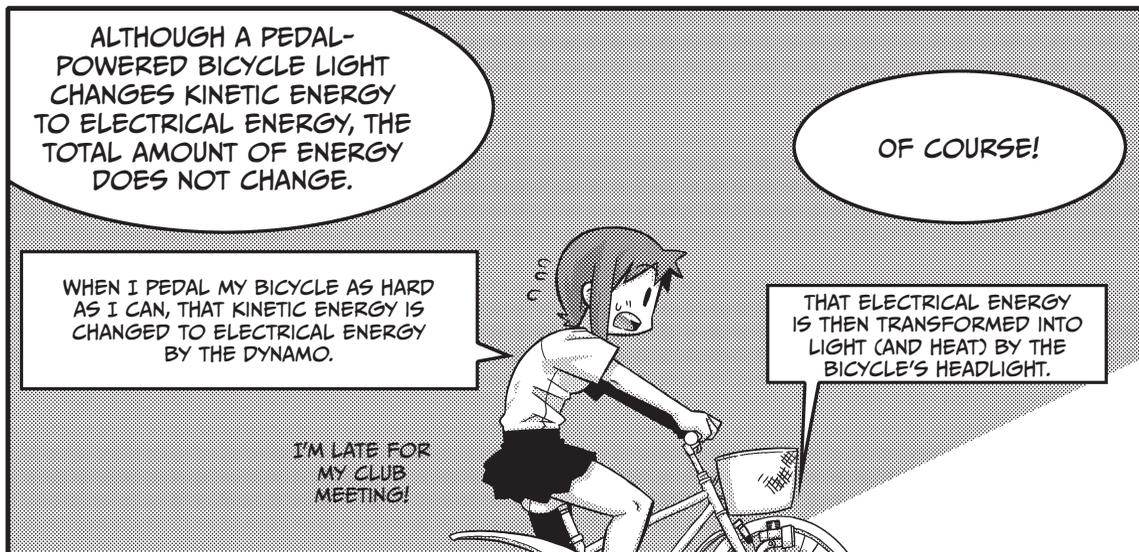
HOWEVER, THE TOTAL MASS INSIDE THE FLASK BEFORE AND AFTER THE REACTION DOES NOT CHANGE.

THAT MAKES SENSE!



...AND WHAT ABOUT THE LAW OF CONSERVATION OF ENERGY? WHAT'S THAT?

THE LAW OF CONSERVATION OF ENERGY SAYS THAT ALTHOUGH ENERGY MAY CHANGE FORM IN VARIOUS WAYS, THE TOTAL AMOUNT DOES NOT CHANGE.



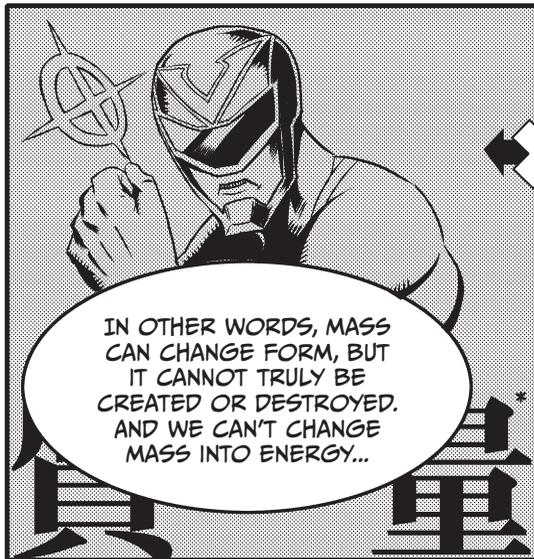
ALTHOUGH A PEDAL-POWERED BICYCLE LIGHT CHANGES KINETIC ENERGY TO ELECTRICAL ENERGY, THE TOTAL AMOUNT OF ENERGY DOES NOT CHANGE.

OF COURSE!

WHEN I PEDAL MY BICYCLE AS HARD AS I CAN, THAT KINETIC ENERGY IS CHANGED TO ELECTRICAL ENERGY BY THE DYNAMO.

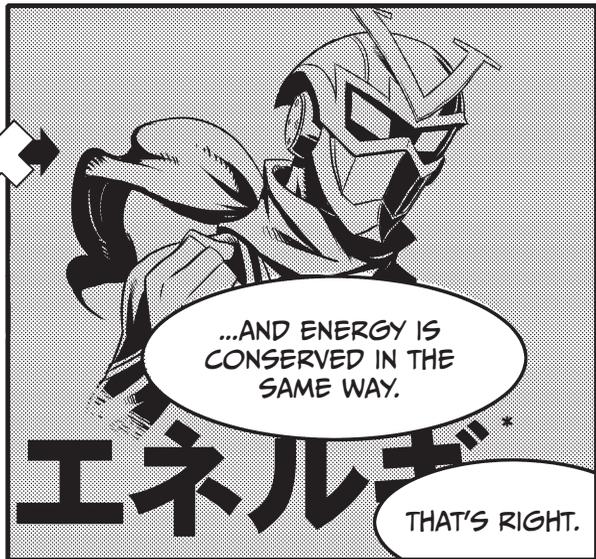
THAT ELECTRICAL ENERGY IS THEN TRANSFORMED INTO LIGHT (AND HEAT) BY THE BICYCLE'S HEADLIGHT.

I'M LATE FOR MY CLUB MEETING!



IN OTHER WORDS, MASS CAN CHANGE FORM, BUT IT CANNOT TRULY BE CREATED OR DESTROYED. AND WE CAN'T CHANGE MASS INTO ENERGY...

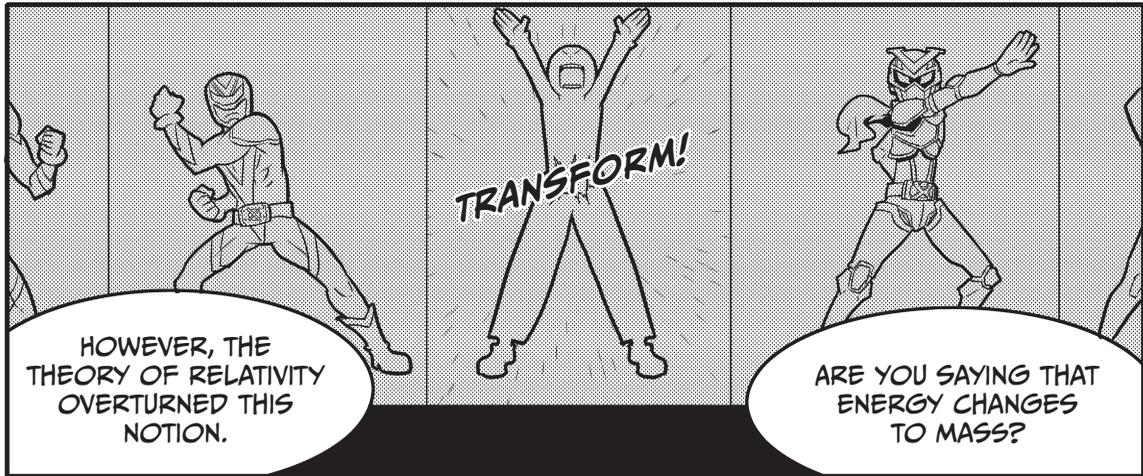
\* MASS



...AND ENERGY IS CONSERVED IN THE SAME WAY.

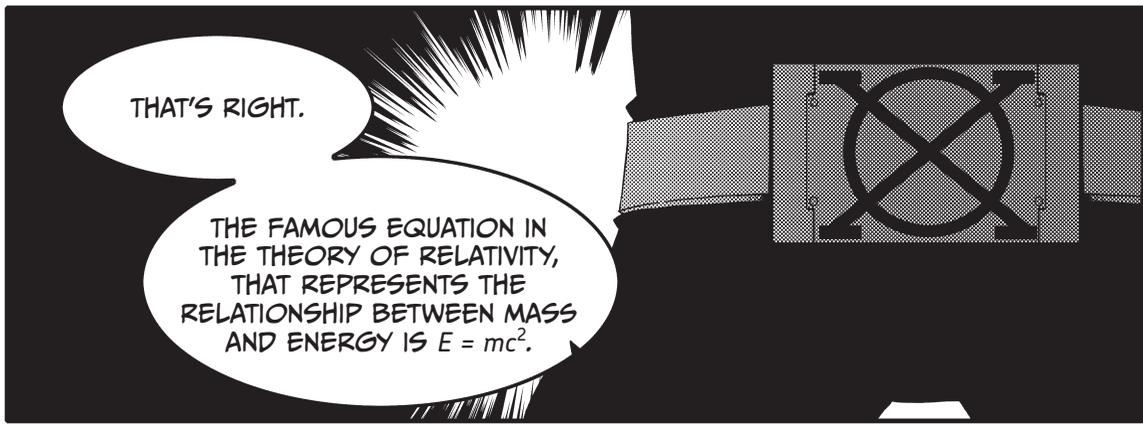
THAT'S RIGHT.

\* ENERGY



HOWEVER, THE THEORY OF RELATIVITY OVERTURNED THIS NOTION.

ARE YOU SAYING THAT ENERGY CHANGES TO MASS?



THAT'S RIGHT.

THE FAMOUS EQUATION IN THE THEORY OF RELATIVITY, THAT REPRESENTS THE RELATIONSHIP BETWEEN MASS AND ENERGY IS  $E = mc^2$ .

WHAT IN THE WORLD  
ARE  $E$ ,  $m$ , AND  $c$ ?



$E$  IS ENERGY.  
 $m$  IS MASS.  
 $c$  IS THE SPEED OF  
LIGHT.

THE EQUATION SAYS THAT  
"ENERGY EQUALS MASS  
MULTIPLIED BY THE SPEED  
OF LIGHT SQUARED."

ENERGY

$E$

SQUARED

$2$

MASS

$m$

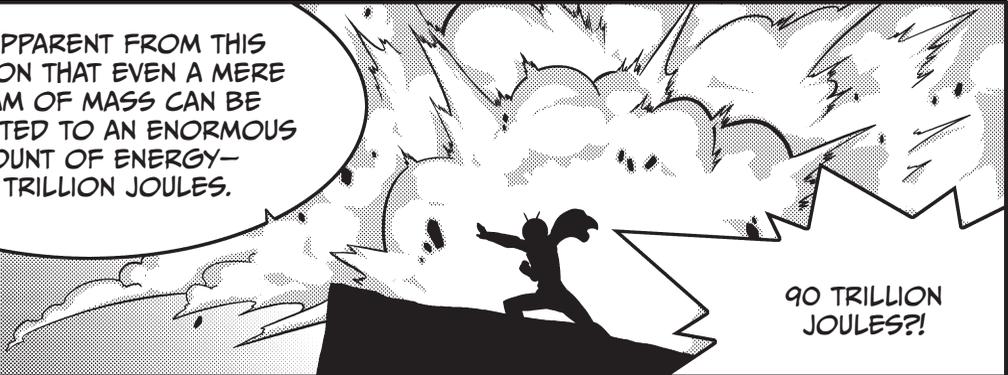
SPEED OF LIGHT

$c$

OF COURSE!  
IT'S THE BELT OF  
TRANSMOGRIFICATION!

WE'RE NOT TALKING  
ABOUT A BELT!

IT'S APPARENT FROM THIS  
EQUATION THAT EVEN A MERE  
1 GRAM OF MASS CAN BE  
CONVERTED TO AN ENORMOUS  
AMOUNT OF ENERGY—  
90 TRILLION JOULES.



90 TRILLION  
JOULES?!

1 GRAM IS ABOUT THE WEIGHT OF A 1 YEN COIN,\* ISN'T IT?

BUT EVEN I DON'T REALLY HAVE CLUE WHAT 90 TRILLION JOULES IS.

MY ENEMY!

HA HA HA! YOU SEEM TO BE ONE STEP BEHIND!

I'VE FROZEN THEM COMPLETELY!

THAT'S RIGHT. FOR EXAMPLE, ASSUME THAT THERE ARE 705 25-METER POOLS FILLED WITH ICE AT  $-10^{\circ}\text{C}$ ...

\* A US PENNY WEIGHS ABOUT 2.5 GRAMS.

...IT'S THE AMOUNT OF ENERGY NEEDED TO HEAT THIS ICE AND TURN IT INTO WATER IN AN INSTANT.

POWER BLAST!

IT TAKES A TREMENDOUS AMOUNT OF ENERGY TO MELT ALL THAT ICE.

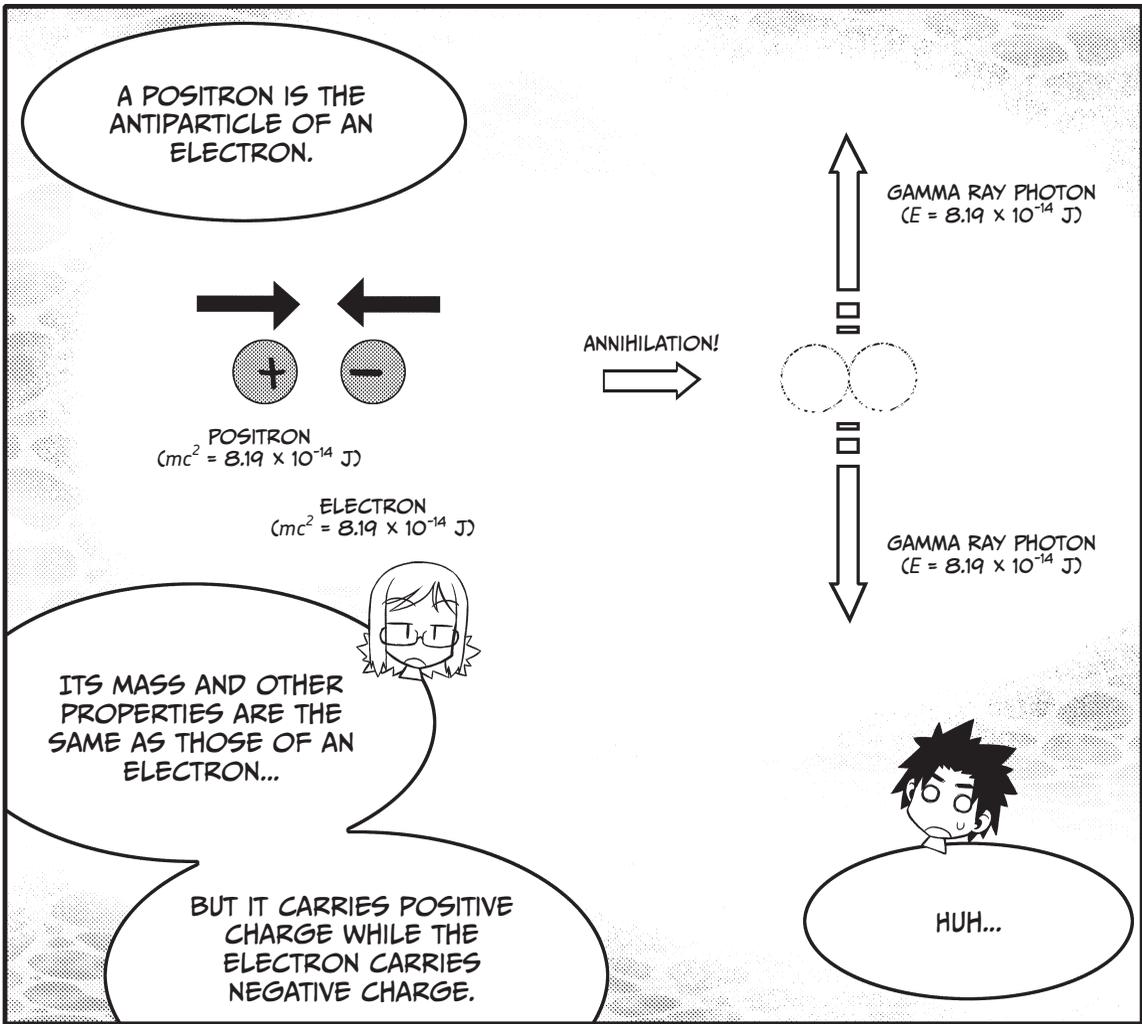
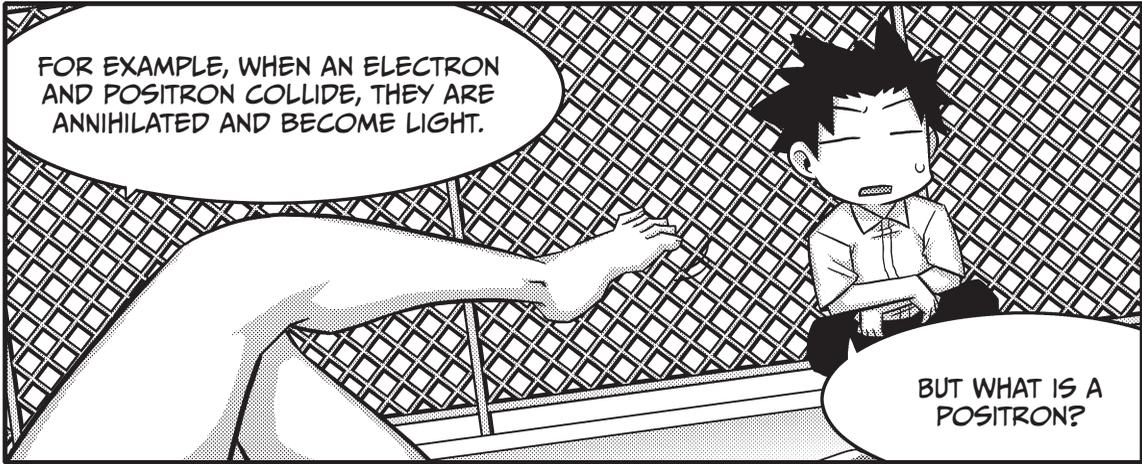
BUT WHAT DO YOU ACTUALLY NEED TO DO TO CHANGE MATTER INTO ENERGY?

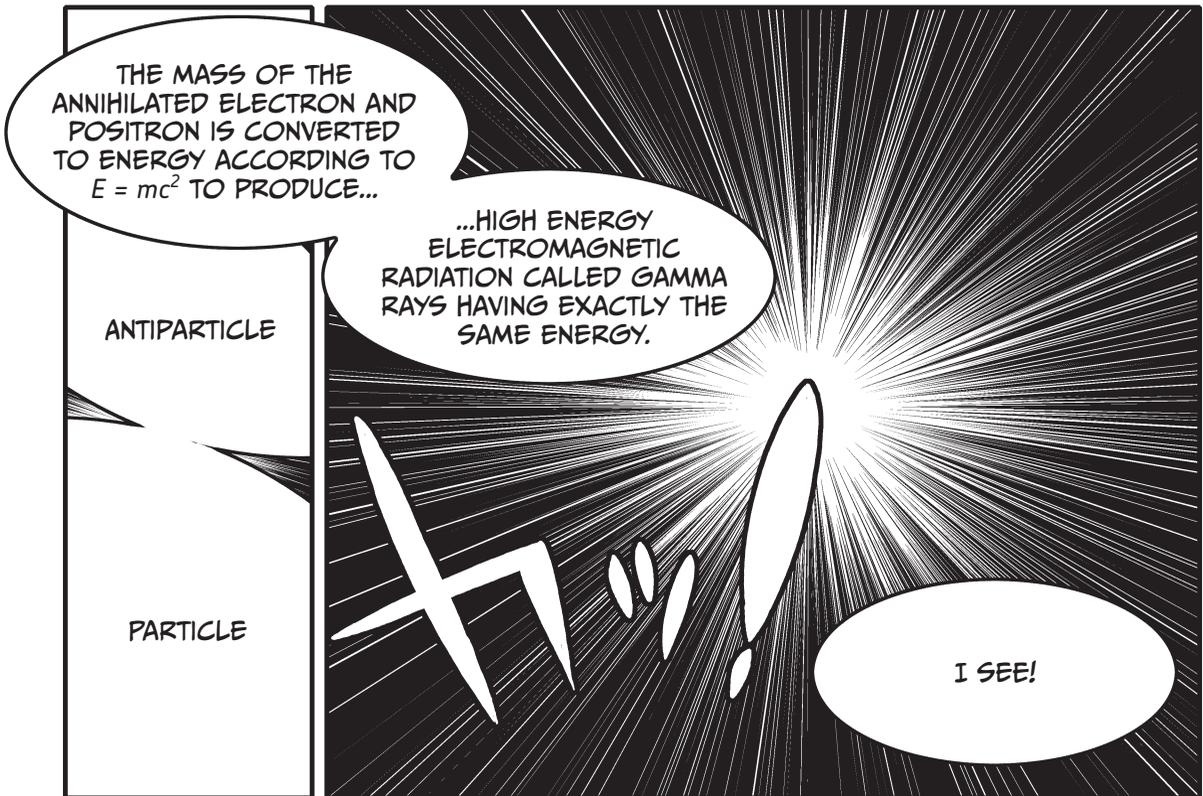
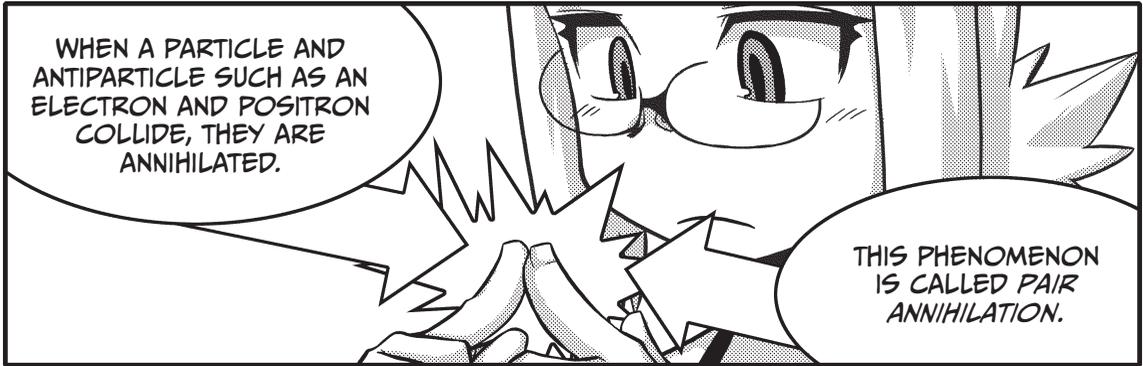
CHANGING 1 GRAM OF MATTER ENTIRELY INTO ENERGY IS UNREALISTIC...

THE END

BUT IN THE SUBATOMIC WORLD, MATTER IS ANNIHILATED AND BECOMES ENERGY AS AN EVERYDAY OCCURRENCE.

IS THAT SO?







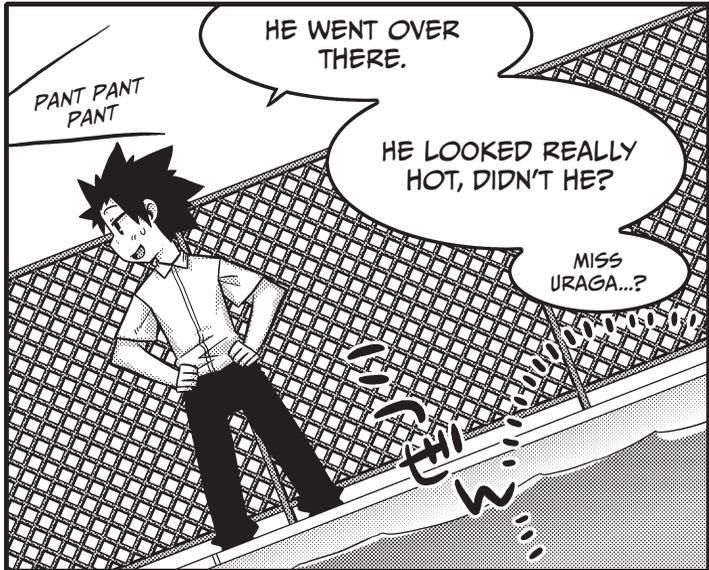
HMMM...I FEEL LIKE I UNDERSTAND IT MORE NOW.

PANT PANT PANT



HEY! ISN'T THAT THE VICE PRINCIPAL?

PANT PANT



PANT PANT PANT

HE WENT OVER THERE.

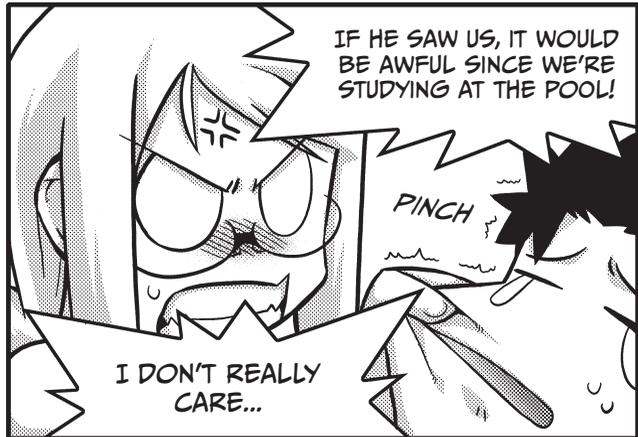
HE LOOKED REALLY HOT, DIDN'T HE?

MISS URAGA...?



IS HE GONE YET?

OMIGOD! YOU LOOK CRAZY HIDING LIKE THAT.



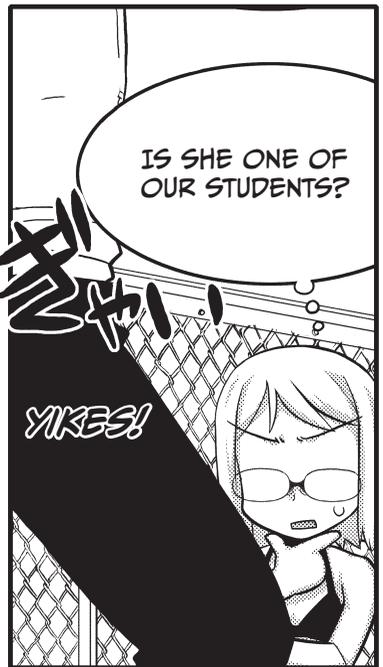
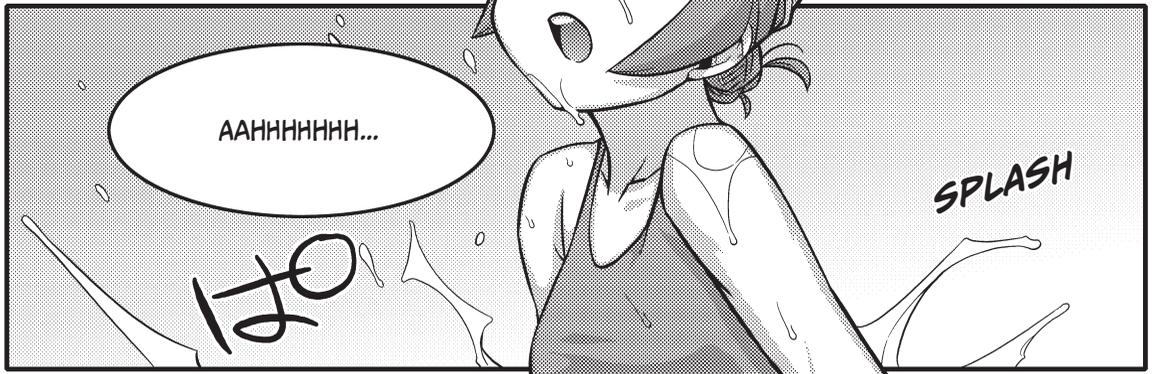
IF HE SAW US, IT WOULD BE AWFUL SINCE WE'RE STUDYING AT THE POOL!

PINCH

I DON'T REALLY CARE...



SPLISH.



# USING AN EQUATION TO UNDERSTAND LENGTH CONTRACTION (LORENTZ CONTRACTION)

Let's use an equation to see how length contracts.

In this case, let's assume that a rocket is flying at a constant velocity  $v$  (see Figure 3-1).

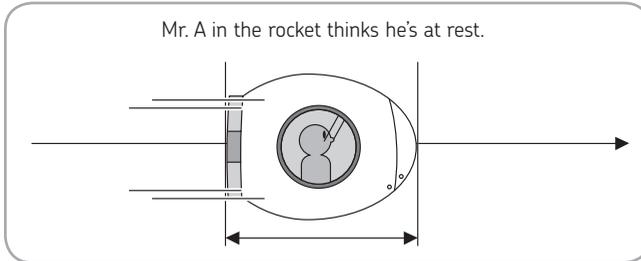


Figure 3-1: A person riding in the rocket measures the positions of the front and back ends of the rocket.

When the person riding in the rocket measures the positions of the front and back ends of the rocket, he finds the front end is at position  $x'_2$ , and the back end is at position  $x'_1$ . Therefore, the rocket's length is  $l_0 = x'_2 - x'_1$ .

Now what happens if this situation is observed from outside the rocket, for example, from a space station as in Figure 3-2?

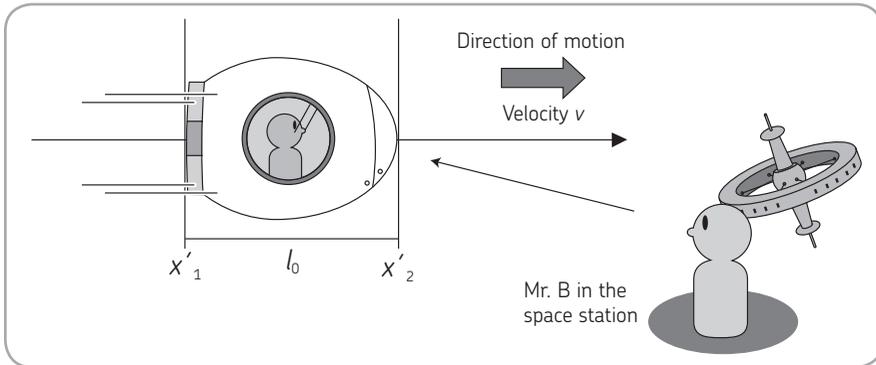


Figure 3-2: The rocket viewed from a space station

To calculate the contraction in the length of a rocket as it moves past an observer at close to the speed of light, let's consider two points in the rocket's frame of reference:  $x'_1$  at the front of the ship and  $x'_2$  at the back of the ship. Using the Lorentz transformation, introduced in "Wait a Second—What Happens with the Addition of Velocities?" on page 48, we can calculate how an observer who watches the ship pass measures the points at the front  $x_1$  and back of the ship  $x_2$ , in his reference frame. The length that the observer on the outside of the ship measures will be shorter than length that the astronaut measures. This effect, *relativistic length contraction*, comes from the contraction of space at speeds close to the speed of light.

Using the Lorentz transformation  $x' = \frac{x - vt}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$ , we calculate the following positions:

$$x'_1 = \frac{x_1 - vt_1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$x'_2 = \frac{x_2 - vt_2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

If we let  $l = x_2 - x_1$  represent the rocket's length as observed from outside the rocket, then since

$$l_0 = x'_2 - x'_1 = \frac{x_2 - vt_2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} - \frac{x_1 - vt_1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{(x_2 - x_1) - (t_2 - t_1)v}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

is measured at the same time,  $t_1 - t_2 = 0$  because  $t_2 = t_1$ , so  $l_0$  is calculated as follows:

$$l_0 = \frac{(x_2 - x_1) - (t_2 - t_1)v}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{(x_2 - x_1)}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{l}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

The length of the spaceship measured from outside the spaceship,

$$l = l_0 \sqrt{1 - \left(\frac{v}{c}\right)^2}$$

is therefore less than the length  $l_0$  of the same spaceship measured from inside the ship. We know this to be true because the coefficient

$$\sqrt{1 - \left(\frac{v}{c}\right)^2} < 1$$

due to the fact that the speed of the spaceship must be slower than the speed of light ( $v < c$ ).

## MUONS WITH EXTENDED LIFE SPANS

Our discussion of time slowing down and length contracting is not just a theoretical proposition. The slowing of time is observed every day.

High-energy elementary particles called *cosmic rays* are raining down on Earth all day, every day. When those cosmic rays collide with molecules in Earth's upper atmosphere, muons are generated with a certain probability. A *muon* is a type of elementary particle that is similar to an electron. The life span of a muon is approximately 2 millionths of a second in a laboratory on the ground at rest. Therefore, when a muon is produced in the upper atmosphere, several tens to several hundreds of kilometers from the ground, it would fly only  $300,000 \text{ km/s} \times 2/1,000,000 \text{ s} = 0.6 \text{ km}$ , even if it were flying at a velocity extremely close to the speed of light. Based on these calculations, it should not reach the surface of Earth. But muons are observed on the surface of Earth! This seemingly impossible event occurs because the muon's life span is extended according to the special theory of relativity (see Figure 3-3). The extension of the lifetime of muons by time dilation has been verified in the laboratory by accelerating muons to near the speed of light.

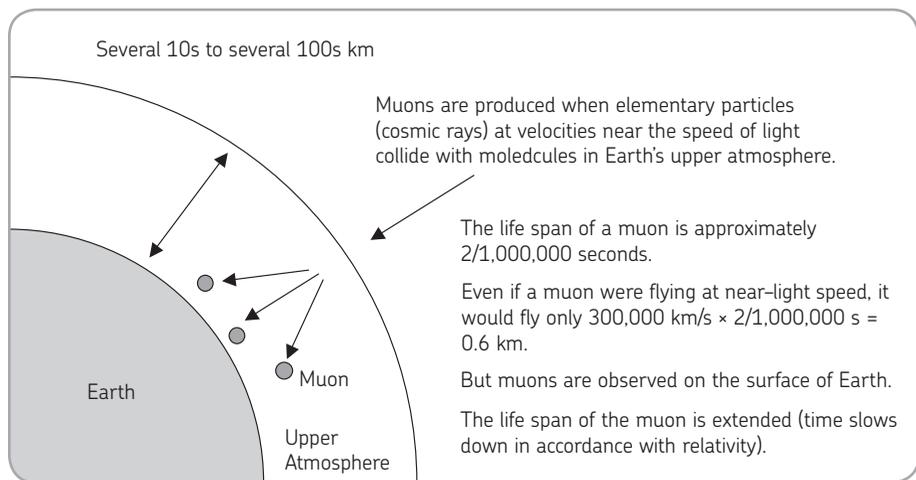


Figure 3-3: Muon life span

Let's now apply the concept of length contraction to the example of the muon. In the reference frame of the muon, the muon's lifetime is not extended—it is still 2 millionths of a second. From the reference frame of the muon, Earth is rushing toward the particle at close to the speed of light. The length between Earth and the muon contracts, however, as shown in Figure 3-4. And because the distance between Earth and the muon contracts, the muon reaches the planet's surface within its lifetime

Both the dilation of time from Earth's perspective and the contraction of length from the muon's perspective are consistent. In this way, the dilation of time and the contraction of length change together according to the theory of relativity.

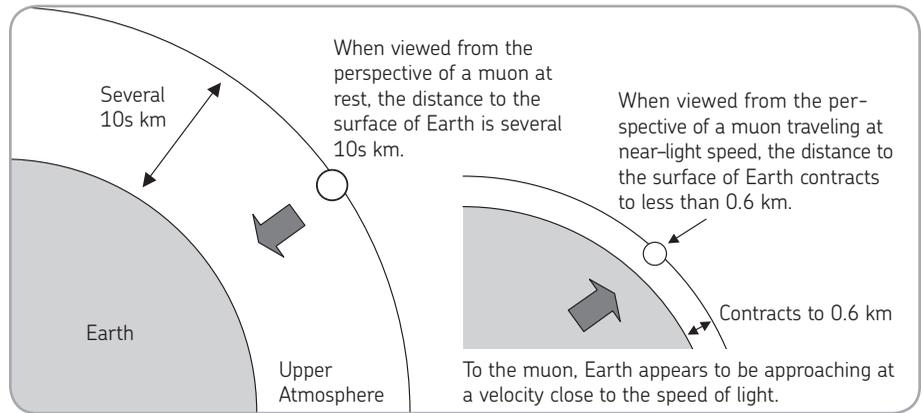


Figure 3-4: Distance contracts as well as time.

## MASS WHEN MOVING

Now let's consider how the mass of an object is related to its velocity according to relativity. Let's start by reviewing the laws of motion. Before relativity was understood, the Galilean transformation and Newton's law of motion were used to describe motion.

### GALILEAN TRANSFORMATION

The Galilean transformation describes the relationship between coordinate systems moving at velocity  $v$ :

$$x' = x - vt \text{ and } t' = t$$

where  $x'$  and  $t'$  represent position and time, respectively, in one system and  $x$ ,  $v$ , and  $t$  represent position, velocity, and time, respectively, in the other system.

### NEWTON'S SECOND LAW OF MOTION

Newton's second law of motion is represented as follows:

$$f = ma = m \frac{d^2x}{dt^2},$$

where  $f$  represents force,  $m$  represents mass,  $a$  represents acceleration, and acceleration can be considered the second derivative of displacement with respect to time:

$$a = \frac{d^2x}{dt^2}$$

According to Galileo's principle of relativity, the laws of physics operate exactly the same way, whether measured while at rest or while moving. In other words, whether you toss a ball in the air inside an elevator that is at rest or inside an elevator that is moving at a constant velocity, the ball will move up and down and return to your hand in the same way.

Now, let's look at the laws of motion in two different reference frames and verify that the laws of physics do not change when we don't take into account relativity. We'll look at the laws of motion in a reference frame that is at rest, in which the position of the ball is measured  $x$ , and in the reference frame of the moving elevator, in which the position of the ball is measured  $x'$  (see Figure 3-5).

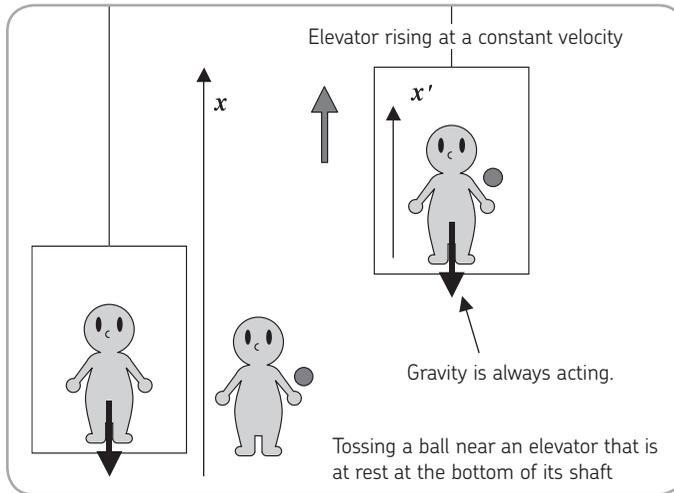


Figure 3-5: Elevator moving at a constant velocity

In this case, the velocity of the ball, which is moving in the  $x'$  direction inside the elevator, is given by

$$\frac{dx'}{dt'}$$

If we substitute the Galilean transformation  $x' = x - vt$  here, we obtain the following:

$$\frac{dx'}{dt'} = \frac{d}{dt'}(x - vt) = \frac{dx}{dt'} - v \frac{dt}{dt'} = \frac{dx}{dt} - v$$

We used the relationship  $\frac{dt'}{dt} = 1$  here, because  $dt' = dt$ .

If we differentiate again, we obtain the following:

$$\frac{d^2x'}{dt'^2} = \frac{d}{dt'} \left( \frac{dx}{dt} - v \right) = \frac{d^2x}{dt^2}$$

Since the only force acting on the ball here is gravity, if we let  $g$  denote gravity, we have this:

$$g = f = ma = m \frac{d^2x}{dt^2}$$

Note that  $g$  is a force, not acceleration due to gravity in this equation. Now, if we let  $a'$  denote the acceleration inside the elevator, which is moving at a constant velocity, and let  $f'$  denote force, we have

$$m \frac{d^2x}{dt^2} = m \frac{d^2x'}{dt'^2} = ma' = f' = g$$

and the form of the equation of motion is unchanged. Since the form of the equation of motion does not change, the laws of physics remain the same.

Let's consider how the situation described above changes when we consider relativity and replace the Galilean transformation with the Lorentz transformation.

## LORENTZ TRANSFORMATION

$$x' = \frac{x - vt}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

As we have shown in this chapter, time and space become intermixed within the framework of relativity. Therefore, when describing the coordinates of an object, it is not sufficient to give its position in three-dimensional space ( $x, y, z$ )—we must also consider its time ( $t$ ). Because the units of position and time are different (meters and seconds, respectively), we multiply time by the speed of light so that we can describe the coordinate position of an object with four dimensions that all have the same unit: length. The following shows that two reference frames are mutually transformed; note that time and space are transformed together.

$$(ct, x, y, z) \leftrightarrow (ct', x', y', z')$$

If we use this thinking to extend the equation of motion so that its form does not change even when a Lorentz transformation is used, it is apparent that mass, which had been considered constant in Newtonian mechanics, is represented in a form similar to the Lorentz transformation:

$$m = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Here,  $m_0$ , which is called the *rest mass* or *invariant mass*, is the mass measured in a coordinate system at rest ( $v = 0$ ).

## RELATIONSHIP BETWEEN ENERGY AND MASS

In the same way, if we consider energy in a form that matches the Lorentz transformation, it is represented as follows:

$$E = \frac{m_0 c^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

If we substitute the earlier relationship,

$$m = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

we derive the famous relationship between energy and mass:  $E = mc^2$ .

Now when  $|x| \ll 1$ , if we use the approximation  $(1 + x)^a \approx 1 + ax$  under the condition

$$\left(\frac{v}{c}\right)^2 \ll 1$$

(velocity  $v$  is sufficiently small compared with the speed of light), we obtain the following:

$$E = \frac{m_0 c^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = m_0 c^2 \left[1 - \left(\frac{v}{c}\right)^2\right]^{-\frac{1}{2}} \cong m_0 c^2 \left[1 + \frac{1}{2} \left(\frac{v}{c}\right)^2\right] = m_0 c^2 + \frac{1}{2} m_0 v^2$$

The total energy of an object is the sum of its kinetic energy

$$E_k = \frac{1}{2}m_0v^2$$

and its rest energy ( $E = mc^2$ ). This means that even when an object is not moving, it has energy associated with its mass. Rest energy  $E = mc^2$  is similar in form to the kinetic energy in Newtonian mechanics, where

$$E_k = \frac{1}{2}m_0v^2$$

and  $m_0c^2$  is called the *rest energy*.

## DOES LIGHT HAVE ZERO MASS?

The equation that we derived above for the mass of an object in motion,

$$m = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

tells us that as the velocity of an object approaches the speed of light, its energy approaches infinity (see Figure 3-6). Therefore, the only way that light can exist (without having infinite energy) is if its mass is 0.

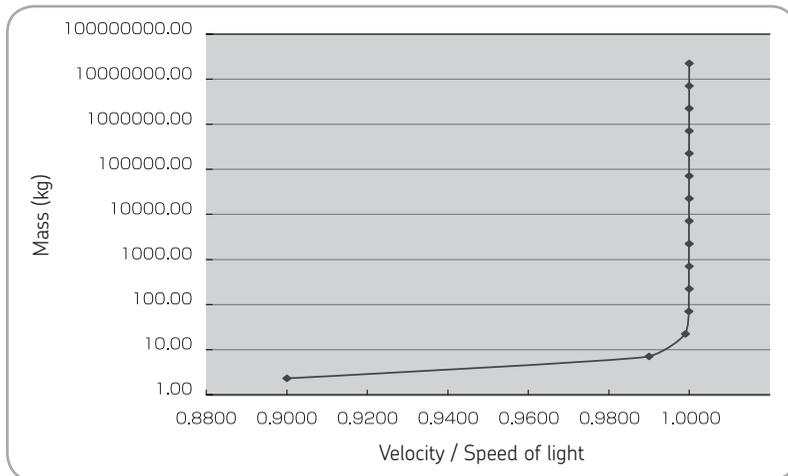
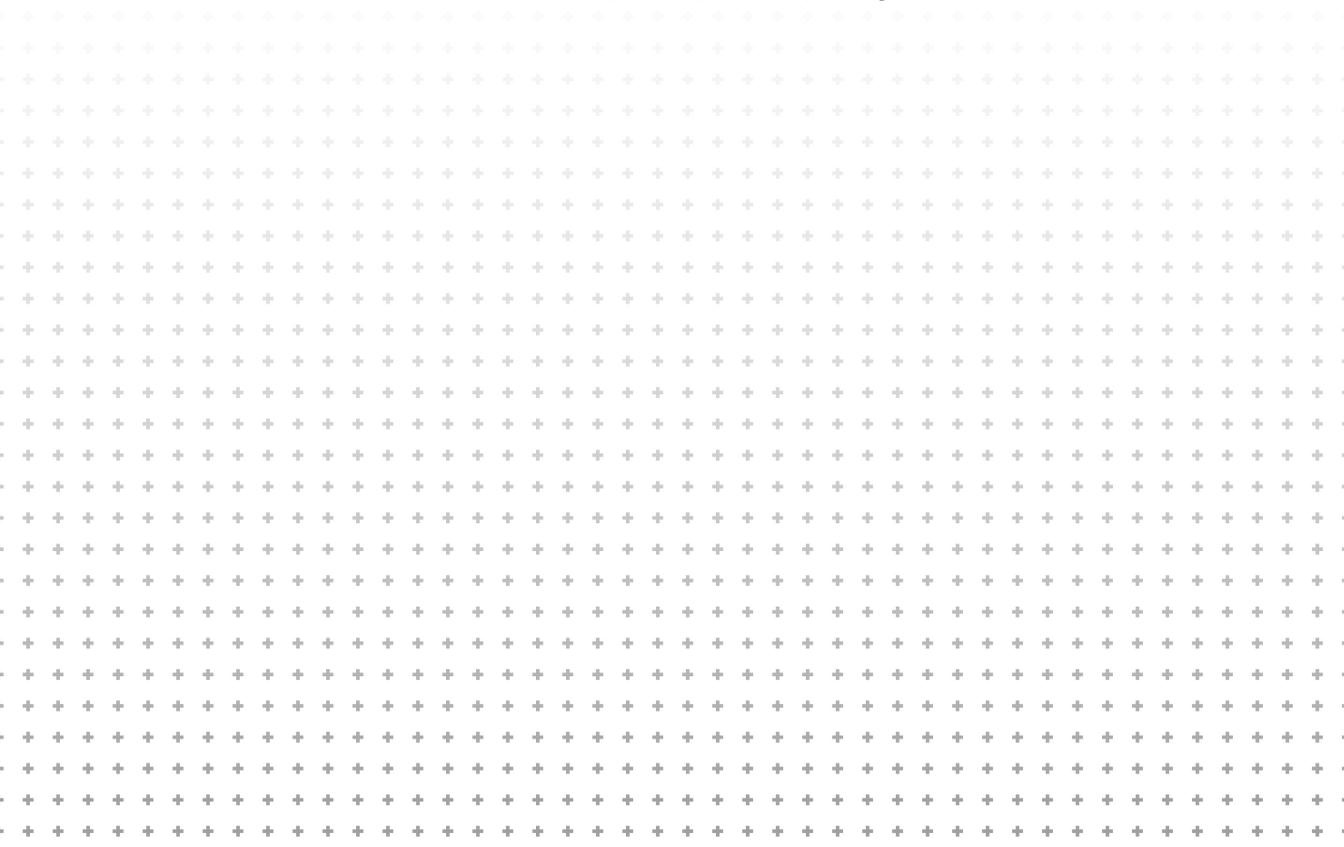


Figure 3-6: Relationship between mass and velocity





# WHAT IS GENERAL RELATIVITY?



HOW'S IT GOING,  
MINAGI?

ARE YOU STUDYING  
RELATIVITY AGAIN  
TODAY?

YEP! HOW ABOUT  
YOU GUYS?

IT'S SO HOT, WE  
REALLY DON'T WANT  
TO DO ANYTHING  
AT ALL!

IT'S NO PICNIC FOR US  
EITHER, MINAGI. WE'VE  
GOT CLUB ACTIVITIES  
AND TONS OF OUR OWN  
HOMEWORK!

HA HA!

THE STUDENT BODY  
PRESIDENT FIGHTING  
FOR ALL STUDENTS!  
AREN'T I INSPIRING?

NOT REALLY.  
BUT THANKS  
TO YOU...

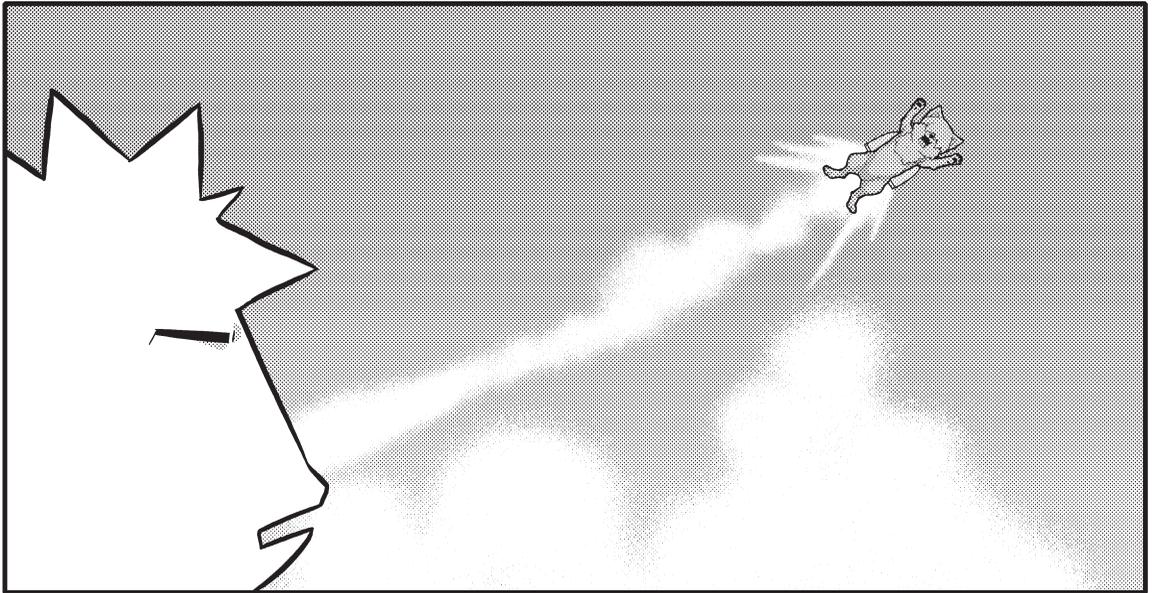
EXCUSE ME

...OUR SUMMER  
VACATION WAS  
SAVED, SO...

LET'S  
GO

WOOSH!

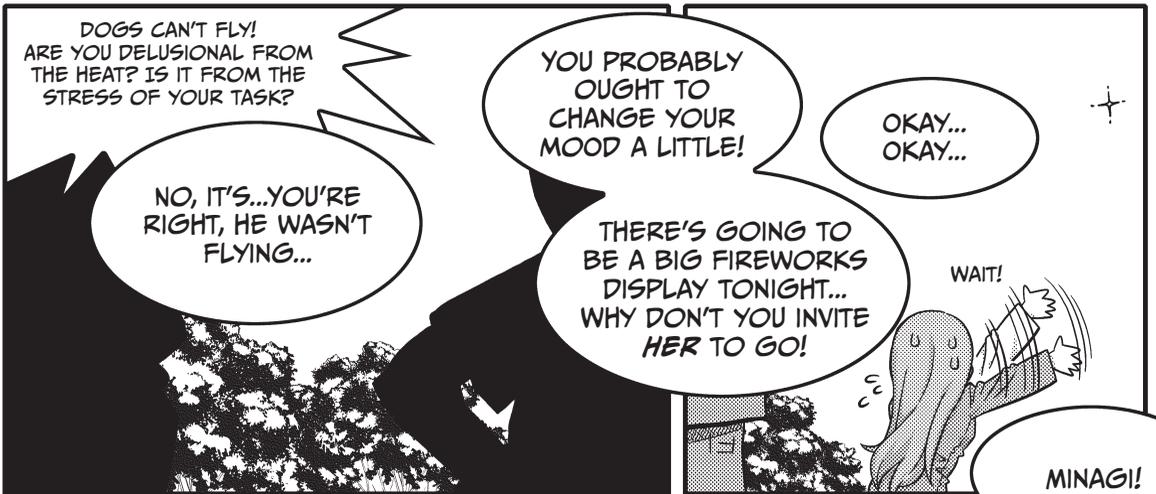
?



YOU MAY THINK I'M CRAZY, BUT JUST LISTEN TO ME FOR A SECOND.

I JUST SAW THE VICE PRINCIPAL FLYING THROUGH THE SKY!

UM, YOU'RE PROBABLY CRAZY.



DOGS CAN'T FLY! ARE YOU DELUSIONAL FROM THE HEAT? IS IT FROM THE STRESS OF YOUR TASK?

NO, IT'S...YOU'RE RIGHT, HE WASN'T FLYING...

YOU PROBABLY OUGHT TO CHANGE YOUR MOOD A LITTLE!

OKAY... OKAY...

THERE'S GOING TO BE A BIG FIREWORKS DISPLAY TONIGHT... WHY DON'T YOU INVITE HER TO GO!

WAIT!

MINAGI!



THAT'S RIGHT. GENERAL RELATIVITY IS MATHEMATICALLY MORE DIFFICULT THAN SPECIAL RELATIVITY.

GENERAL RELATIVITY

SPECIAL RELATIVITY

OBSERVER

OBSERVER

I SAID THIS EVEN IN MY FIRST LECTURE, REMEMBER?

The diagram is divided into two parts. The top part, labeled 'GENERAL RELATIVITY', shows a large sphere representing a massive star. A curved line representing light or a path bends around the star. Two small robot-like figures labeled 'OBSERVER' are positioned at different points along this curved path. The bottom part, labeled 'SPECIAL RELATIVITY', shows three robot-like figures moving in a straight line from left to right, with an arrow indicating their direction of travel.

THEREFORE, I'LL JUST SUPERFICIALLY EXPLAIN WHAT GENERAL RELATIVITY IS ABOUT.

A large hand is shown in profile, pointing its index finger towards the right side of the panel.

GENERAL RELATIVITY SAYS THAT LIGHT WILL BEND AND THE PASSAGE OF TIME WILL SLOW DOWN IN THE VICINITY OF A MASSIVE STAR THAT HAS LARGE GRAVITATIONAL PULL.

LET'S TALK ABOUT THAT!

I'LL DO WHATEVER I CAN TO BE ABLE TO UNDERSTAND THIS!

A character with spiky black hair and a white shirt is shown from the chest up. He has a thoughtful expression with his eyes closed and a small sweat drop on his forehead. He is standing in front of a window with a grid pattern.

1. EQUIVALENCE PRINCIPLE

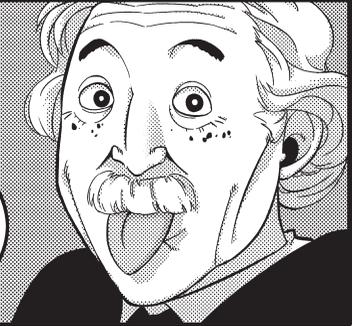
SPECIAL RELATIVITY IS A THEORY THAT HOLDS IN A COORDINATE SYSTEM WITH NO GRAVITY OR ACCELERATION—A SYSTEM THAT IS CALLED AN INERTIAL FRAME.

HOWEVER, THIS KIND OF IDEAL STATE DOES NOT REALLY EXIST.

THAT'S CERTAINLY TRUE!

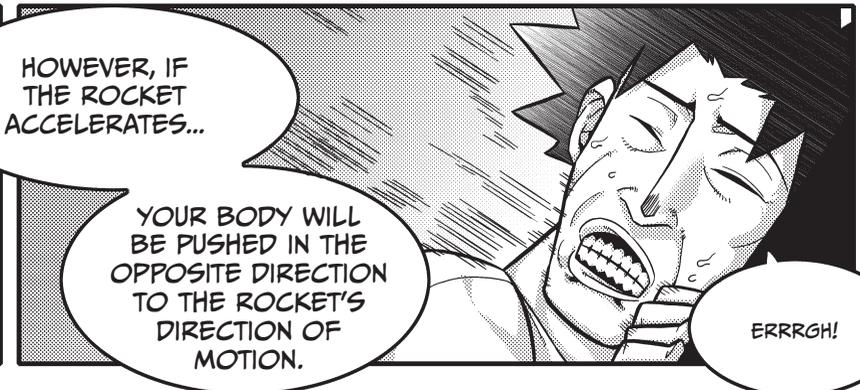
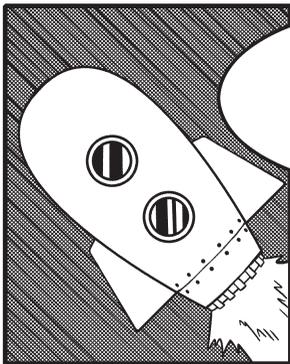
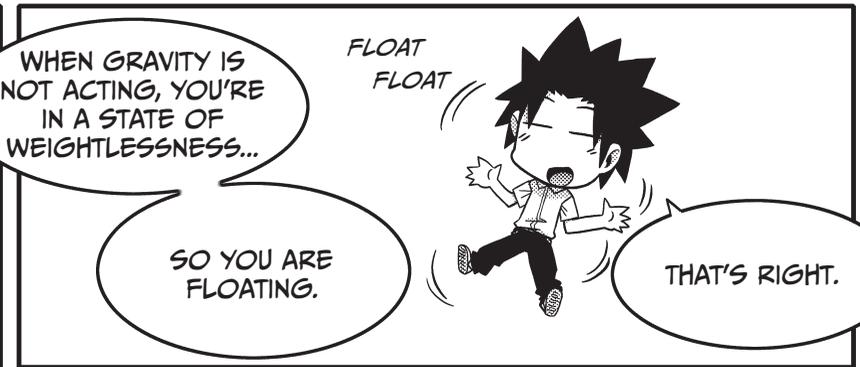
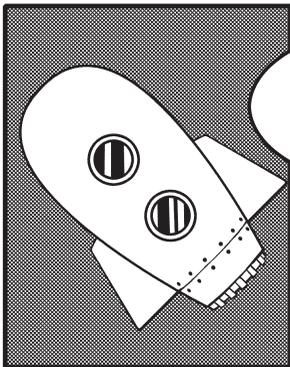
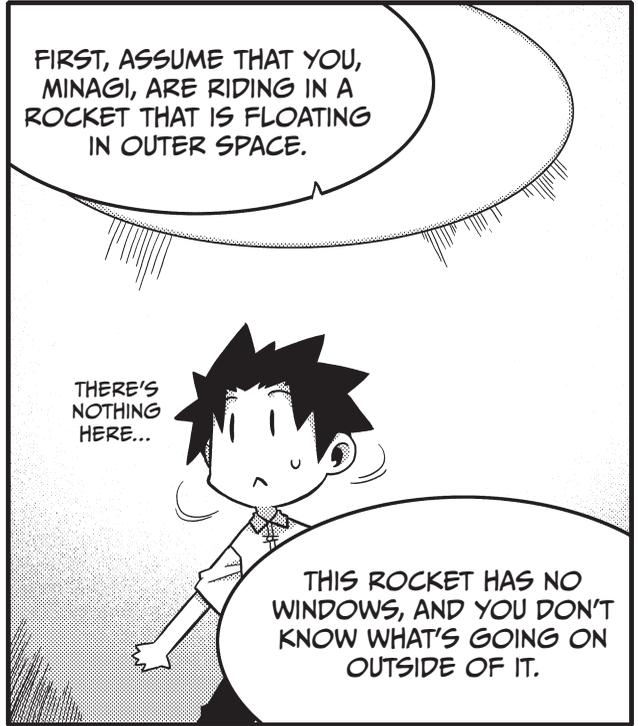
THEREFORE, EINSTEIN THOUGHT HE WOULD INCORPORATE GRAVITY OR ACCELERATION INTO RELATIVITY...

AND CAME UP WITH THE IMPORTANT IDEA CALLED THE EQUIVALENCE PRINCIPLE, WHICH IS THE FOUNDATION OF GENERAL RELATIVITY.



THIS SAYS THAT "THE INERTIAL FORCE ACCOMPANYING ACCELERATED MOTION IS INDISTINGUISHABLE FROM GRAVITY, AND THEREFORE, THEY ARE THE SAME."

**EQUIVALENCE**



IT'S THE SAME FEELING AS THE BACKWARD FORCE YOUR BODY RECEIVES WHEN A TRAIN STARTS TO MOVE, ISN'T IT?

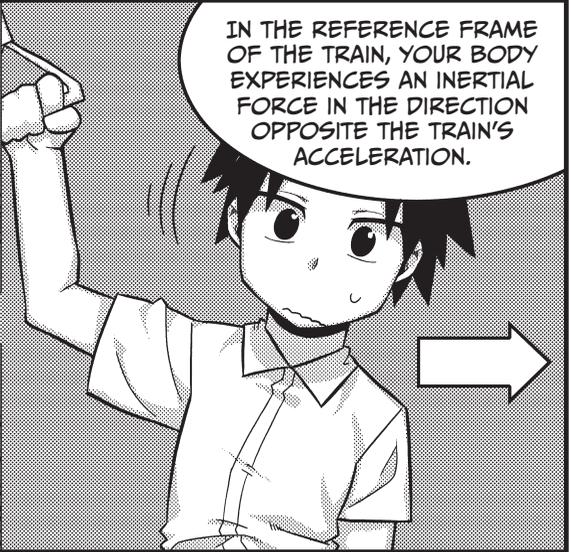
CHUG-A-LUG  
CHUG-A-LUG

OH OH

THAT'S RIGHT. THAT FORCE IS CALLED THE *INERTIAL FORCE*. SINCE YOU ALREADY BROUGHT IT UP, LET'S CONSIDER A TRAIN FOR AN EXAMPLE.

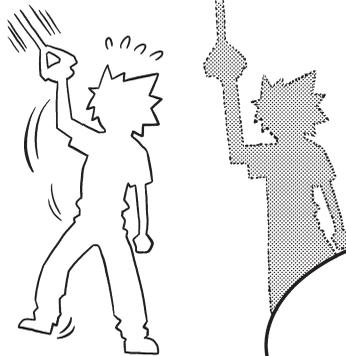
WHEN A TRAIN DEPARTS, IT ACCELERATES IN THE DIRECTION OF MOTION TO MOVE FORWARD.

IN THE REFERENCE FRAME OF THE TRAIN, YOUR BODY EXPERIENCES AN INERTIAL FORCE IN THE DIRECTION OPPOSITE THE TRAIN'S ACCELERATION.



NEWTON'S FIRST LAW STATES THAT OBJECTS AT REST WILL STAY AT REST UNTIL ACTED UPON BY AN OUTSIDE FORCE. WHEN THE TRAIN ACCELERATES, YOU EXPERIENCE A FORCE FROM THE TRAIN'S SEAT ONTO YOUR BACK.

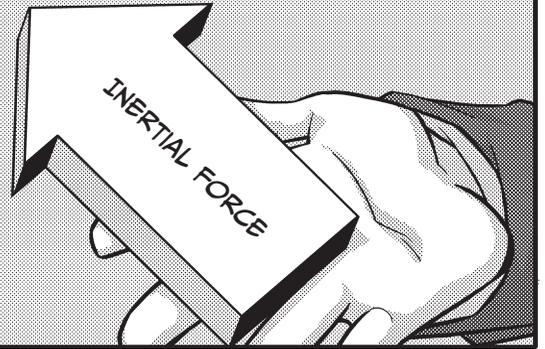
MOVEMENT OF TRAIN



I SEE!

IN OTHER WORDS, THIS HAPPENS BECAUSE YOUR BODY OBEYS THE LAW OF INERTIA.

THE FORCE YOU FEEL AT THAT TIME IS CALLED THE INERTIAL FORCE.



THIS INERTIAL FORCE IS FELT ONLY BY PEOPLE WHO ARE INSIDE THE TRAIN.



CREEEAK

THIS INERTIAL FORCE IS NOT FELT BY A PERSON ON THE PLATFORM OUTSIDE OF THE TRAIN.

SIMILARLY, WHEN THE TRAIN DECELERATES, THE INERTIAL FORCE IS FELT IN THE DIRECTION OPPOSITE TO THE DIRECTION OF DECELERATION.

YOU KNOW THIS SINCE YOUR BODY TUMBLES FORWARD AS IT TRIES TO MAINTAIN ITS VELOCITY.



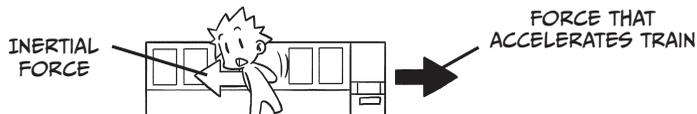
WHEN THE TRAIN ACCELERATES OR DECELERATES, THE INERTIAL FORCE ACTS IN THIS WAY WITH THE SAME MAGNITUDE IN THE OPPOSITE DIRECTION.



TRAIN AT REST



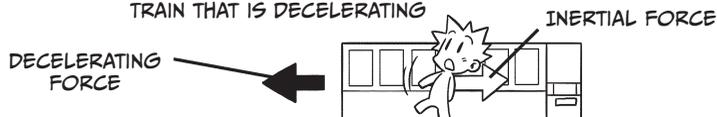
TRAIN THAT IS ACCELERATING



TRAIN THAT IS RUNNING AT CONSTANT VELOCITY



TRAIN THAT IS DECELERATING



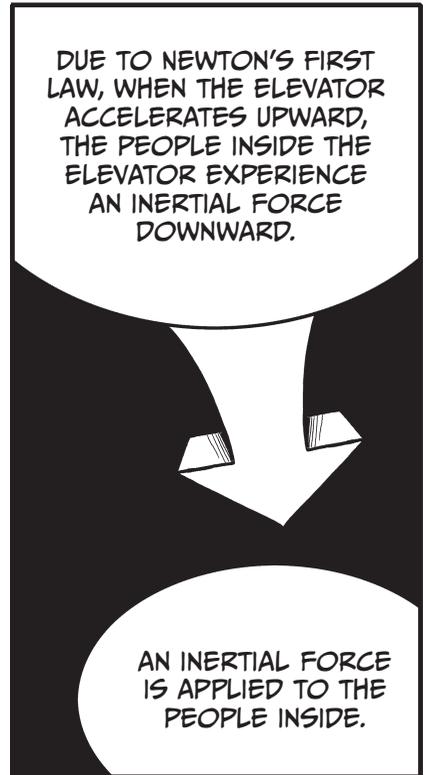
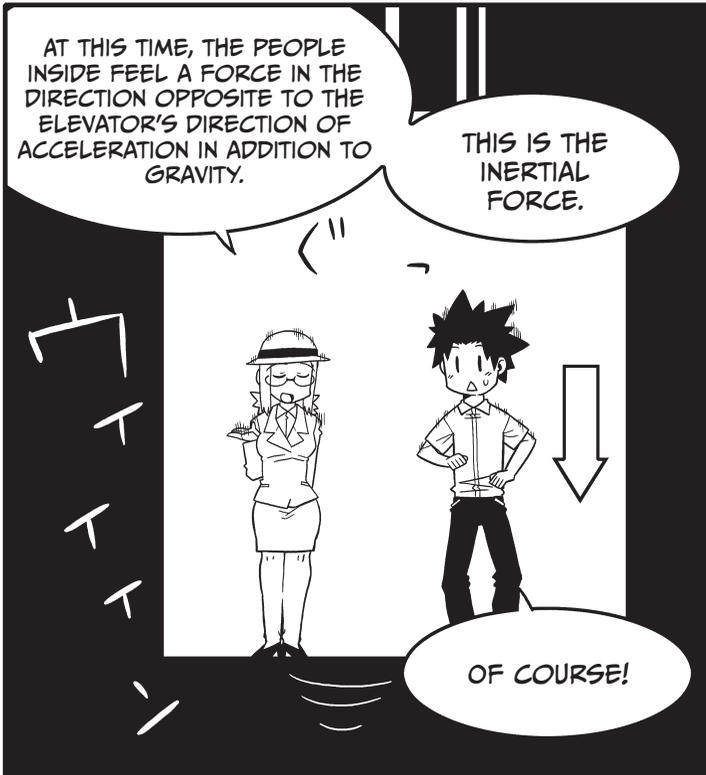
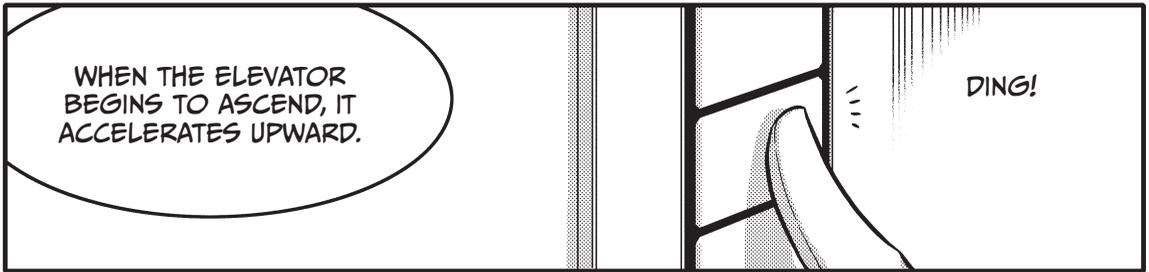
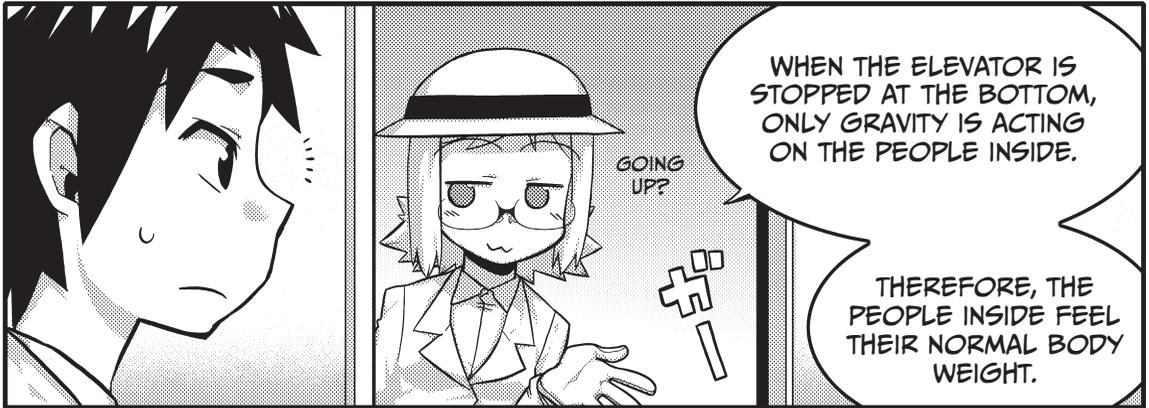
TRAIN AT REST

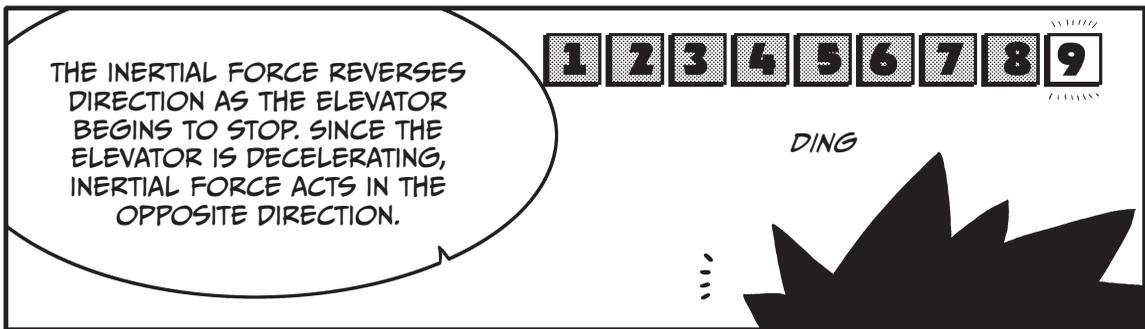
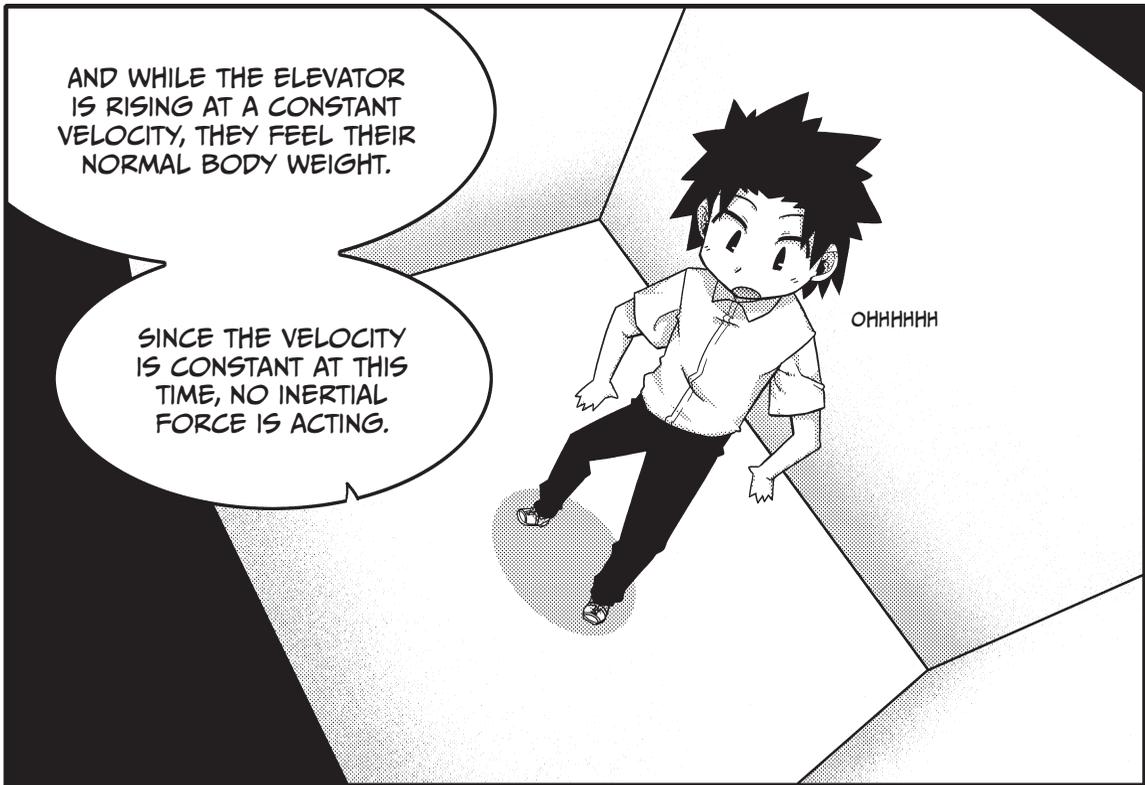


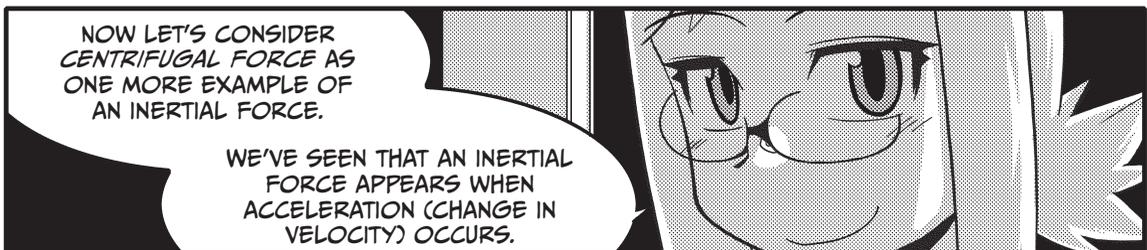
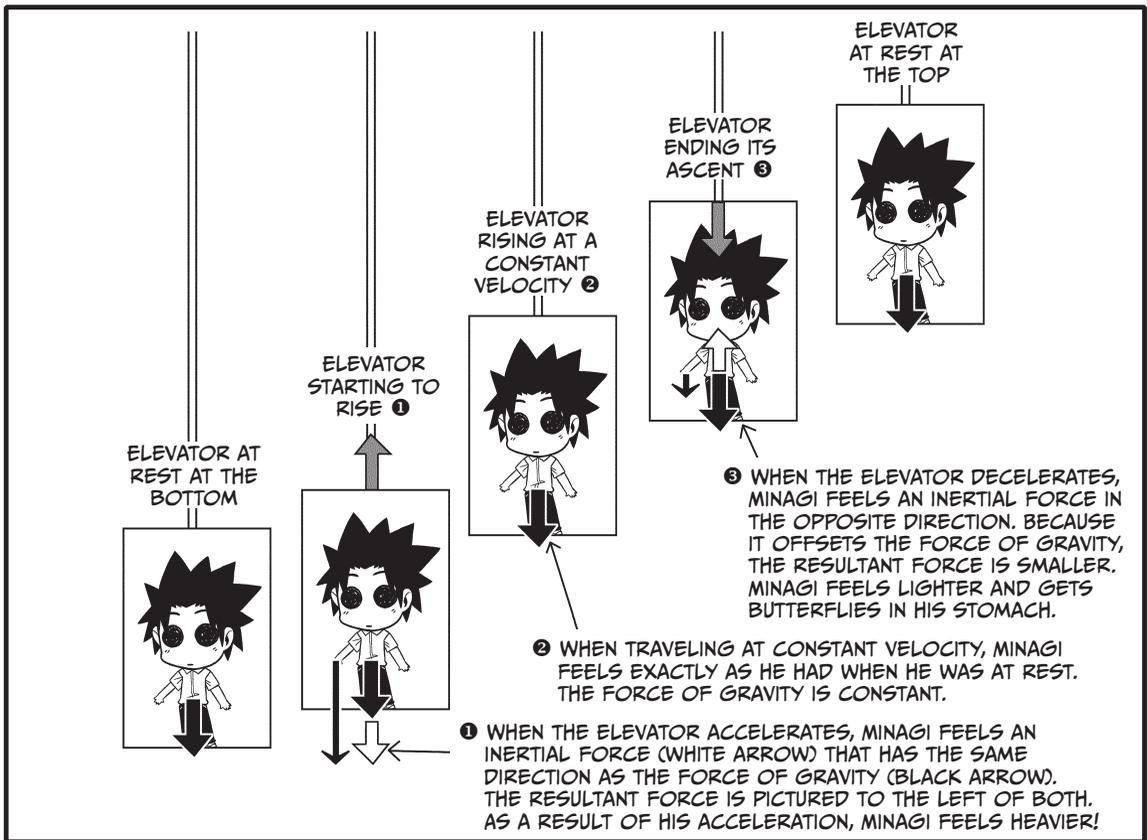
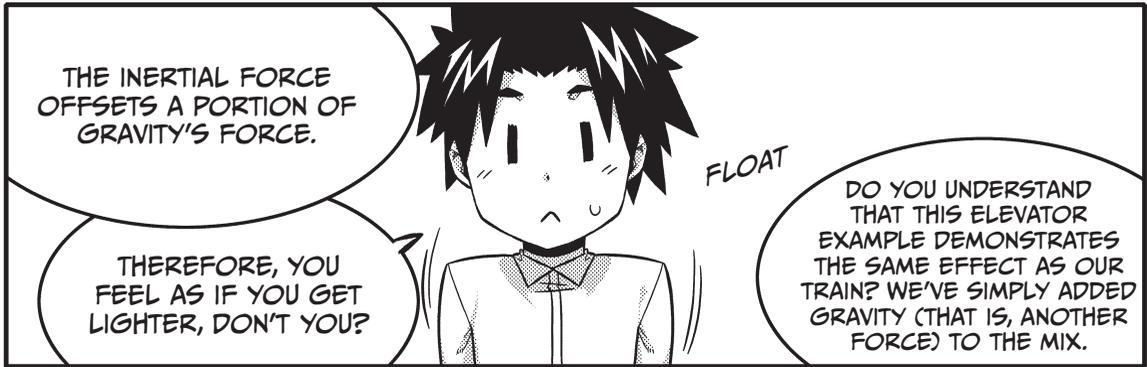
NEXT, LET'S TAKE AN ELEVATOR AS AN EXAMPLE TO CONSIDER THE CASE WHEN GRAVITY IS ALSO ACTING.



AN ELEVATOR?







YOU'VE RIDDEN ON THE SWINGS AT AN AMUSEMENT PARK BEFORE, RIGHT?

WE'RE HERE!

YOU SIT IN A CHAIR AND FLY AROUND AND AROUND. YOU FEEL A FORCE EXERTED ON YOU THAT PUSHES YOU OUTWARD. THIS IS A *CENTRIFUGAL* FORCE, A SPECIFIC KIND OF INERTIAL FORCE.

IS THAT SO?

THE PERSON ON THE SWING HAS A VELOCITY THAT IS CONSTANTLY CHANGING DIRECTION, SUCH THAT THE PERSON TRAVELS IN A CIRCLE.

WHY DO YOU SUPPOSE THAT'S SO? THE PERSON ON THE SWING IS ACCELERATING...

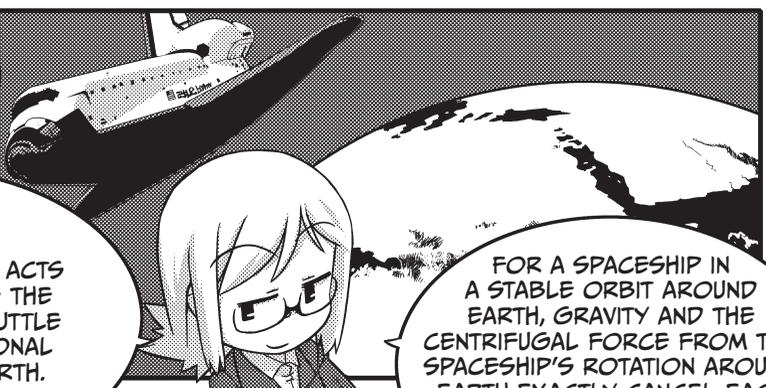
...TOWARD THE CENTER OF THE SWING'S ROTATION.

IF WE THINK ABOUT THE SPACE SHUTTLE...

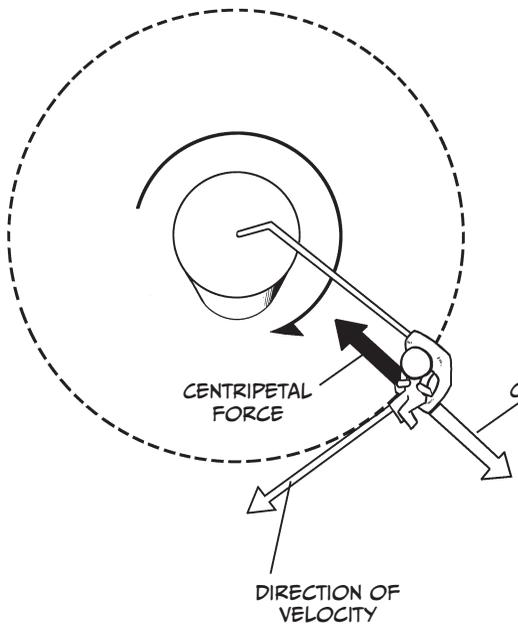
GRAVITY FROM EARTH ACTS ON A PERSON INSIDE THE SHUTTLE, AND THE SHUTTLE UNDERGOES ROTATIONAL MOTION AROUND EARTH.

FOR A SPACESHIP IN A STABLE ORBIT AROUND EARTH, GRAVITY AND THE CENTRIFUGAL FORCE FROM THE SPACESHIP'S ROTATION AROUND EARTH EXACTLY CANCEL EACH OTHER. THIS CREATES A FEELING OF WEIGHTLESSNESS FOR THE ASTRONAUTS.

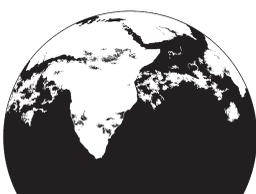
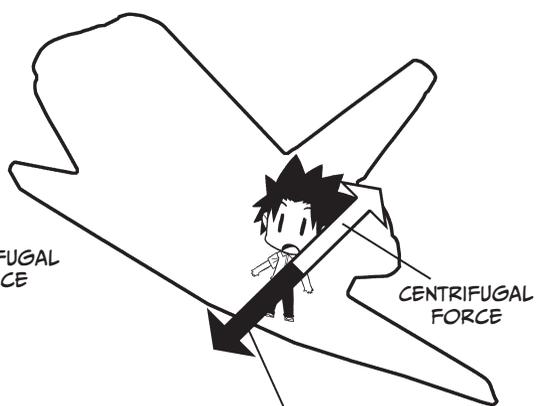
ESSENTIALLY, ALTHOUGH THE PERSON SHOULD MOVE DIRECTLY STRAIGHT AHEAD, HE RECEIVES A FORCE IN THE DIRECTION TOWARD THE CENTER OF ROTATION, WHICH CHANGES HIS VELOCITY.



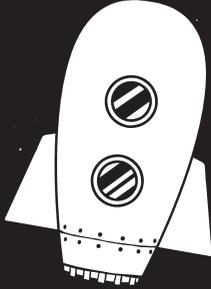
AMUSEMENT PARK SWING RIDE



INSIDE THE SPACE SHUTTLE, THERE IS A STATE OF WEIGHTLESSNESS BECAUSE GRAVITY AND THE CENTRIFUGAL FORCE ARE IN BALANCE.



NOW, LET'S RETURN TO OUR DISCUSSION OF THE ROCKET.



IF A ROCKET IN ZERO GRAVITY ACCELERATES WITH THE SAME ACCELERATION THAT GRAVITY CAUSES OBJECTS TO FALL AT ON EARTH...

**UPWARD**

AM I FLYING?!

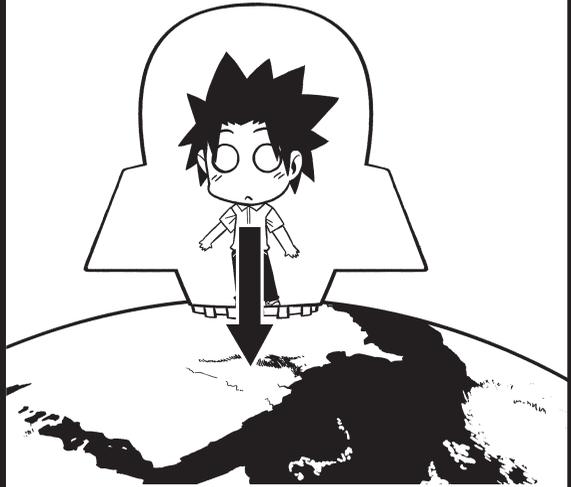
...AN INERTIAL FORCE THAT IS EXACTLY THE SAME AS GRAVITY ON THE GROUND WILL BE APPLIED TO MINAGI.

ACCELERATION OF ROCKET,  $9.8 \text{ M/S}^2$  ↑ ZERO GRAVITY



THE INERTIAL FORCE MAKES IT FEEL AS IF GRAVITY WERE ACTING.

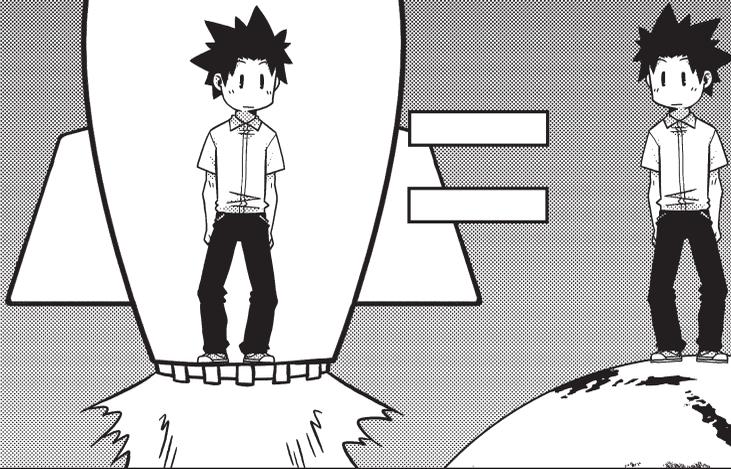
IF YOU COULDN'T SEE OUTSIDE THE ROCKET, YOU'D THINK YOU NEVER LEFT EARTH. THAT'S BECAUSE THE FORCE OF THIS ACCELERATION IS EQUAL TO THE FORCE OF EARTH'S GRAVITY.



YES, I SUPPOSE THAT MAKES SENSE.

INSIDE THE ROCKET, IT IS IMPOSSIBLE TO DISTINGUISH BETWEEN THE EFFECT OF GRAVITY AND THE EFFECT OF THE ROCKET SHIP ACCELERATING.

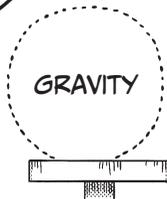
EINSTEIN MADE THIS OBSERVATION AND PROPOSED THE *EQUIVALENCE PRINCIPLE*, WHICH SAYS THAT GRAVITY AND ACCELERATION ARE ACTUALLY THE SAME THING.



IF WE CONSIDER THIS IN REVERSE, GRAVITY CAN ALSO BE ELIMINATED.

POOF

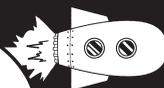
GRAVITY



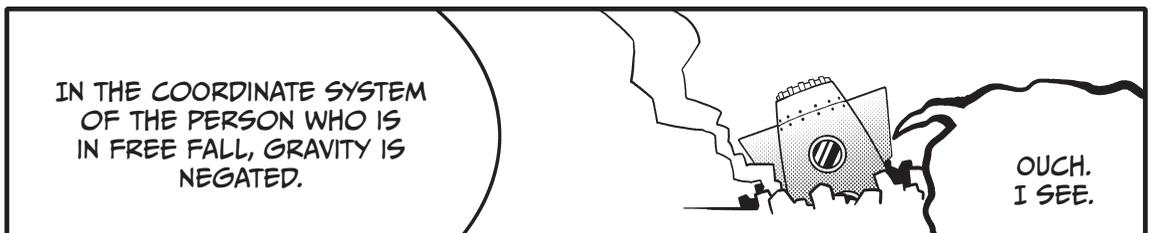
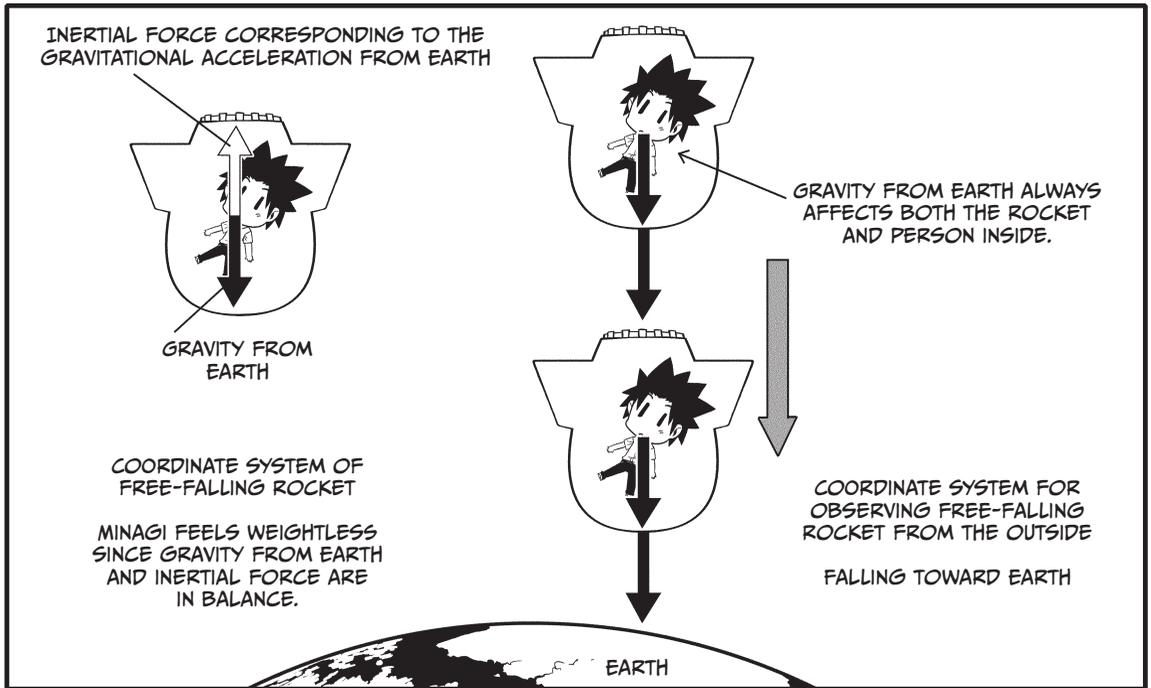
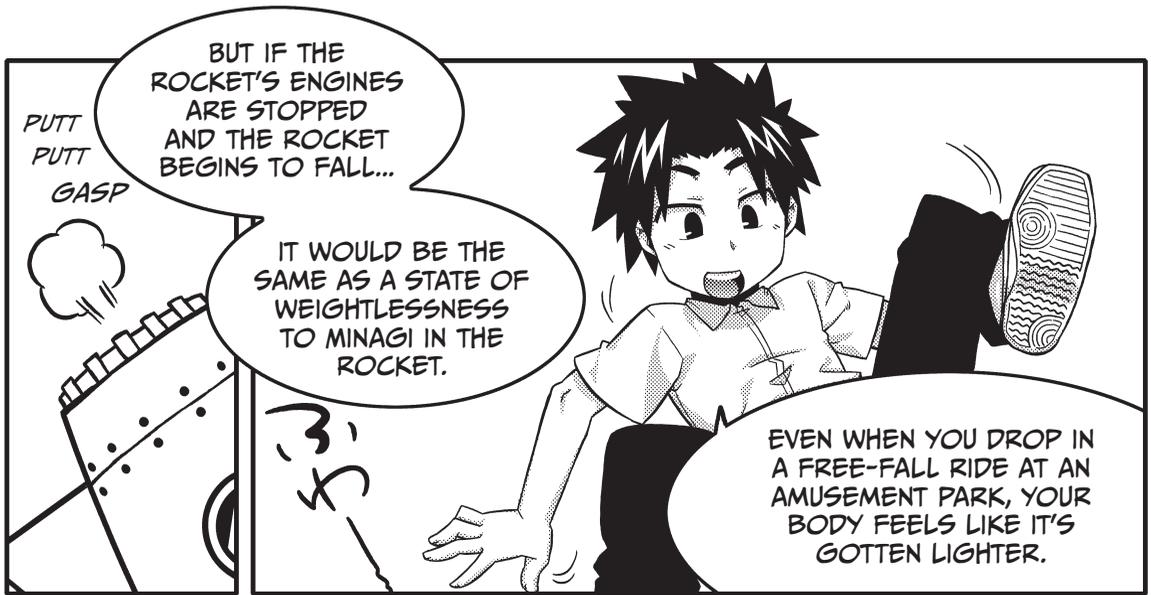
WHAT DO YOU MEAN?

LET'S ASSUME THAT THE ROCKET WE WERE TALKING ABOUT EARLIER ARRIVED AT EARTH.

WHEN IT FLEW INTO EARTH'S GRAVITATIONAL FIELD, GRAVITY WOULD NATURALLY AFFECT MINAGI, WHO IS RIDING IN IT.



THAT'S RIGHT.



2. LIGHT IS BENT BY GRAVITY

GRAVITY IS NEGATED BY THE INERTIAL FORCE INSIDE THE FREE-FALLING ROCKET.

IN OTHER WORDS...

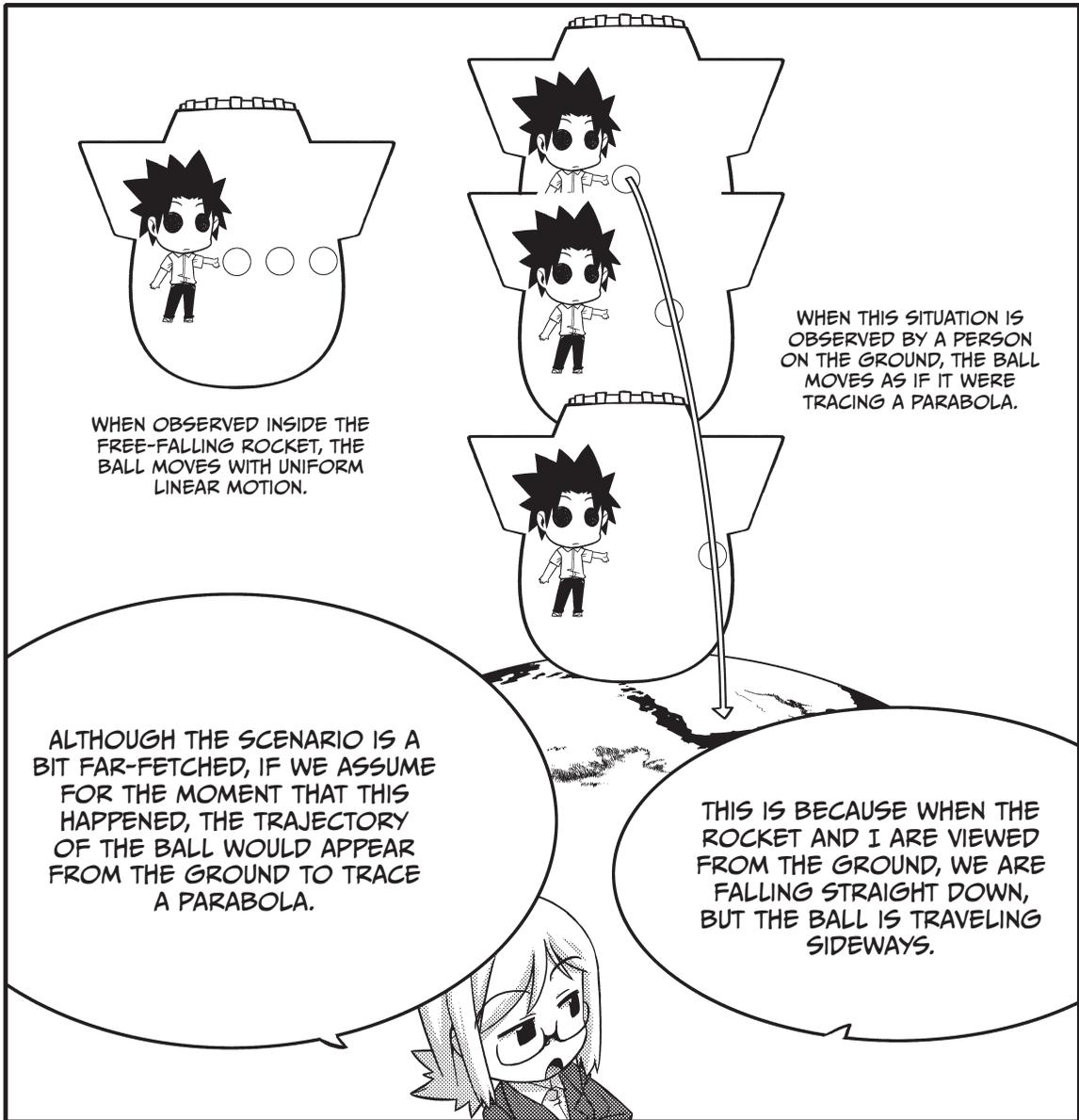
...THE INTERIOR OF THE ROCKET IS CONSIDERED TO BE AN INERTIAL FRAME UNAFFECTED BY GRAVITY.

OKAY.

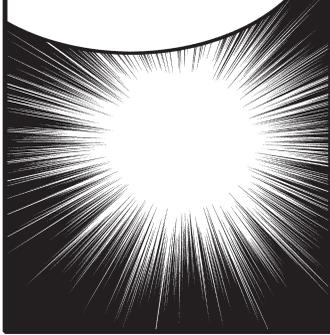
THEREFORE, IF YOU LIGHTLY POKE THE SIDE OF A BALL, THE BALL WILL FLY WITH A FIXED VELOCITY.

THAT'S RIGHT.

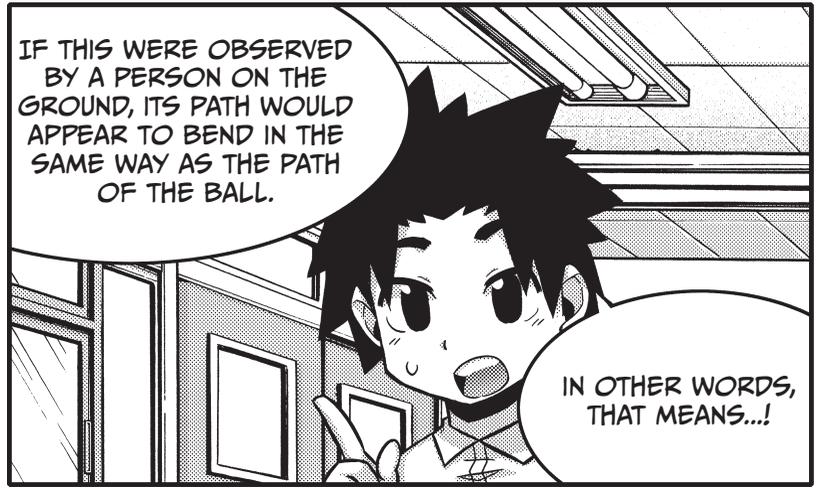
HOWEVER, WHAT WILL HAPPEN WHEN THIS SITUATION IS OBSERVED BY A PERSON ON EARTH?



BUT WHAT HAPPENS  
IF YOU EMIT LIGHT  
INSTEAD OF  
PUSHING A BALL?

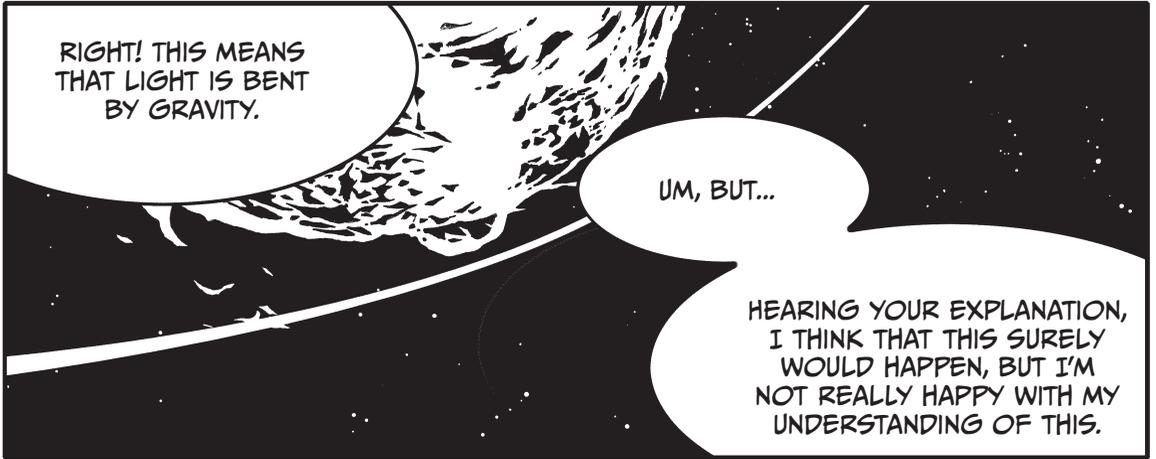


IF THIS WERE OBSERVED  
BY A PERSON ON THE  
GROUND, ITS PATH WOULD  
APPEAR TO BEND IN THE  
SAME WAY AS THE PATH  
OF THE BALL.



IN OTHER WORDS,  
THAT MEANS...!

RIGHT! THIS MEANS  
THAT LIGHT IS BENT  
BY GRAVITY.



UM, BUT...

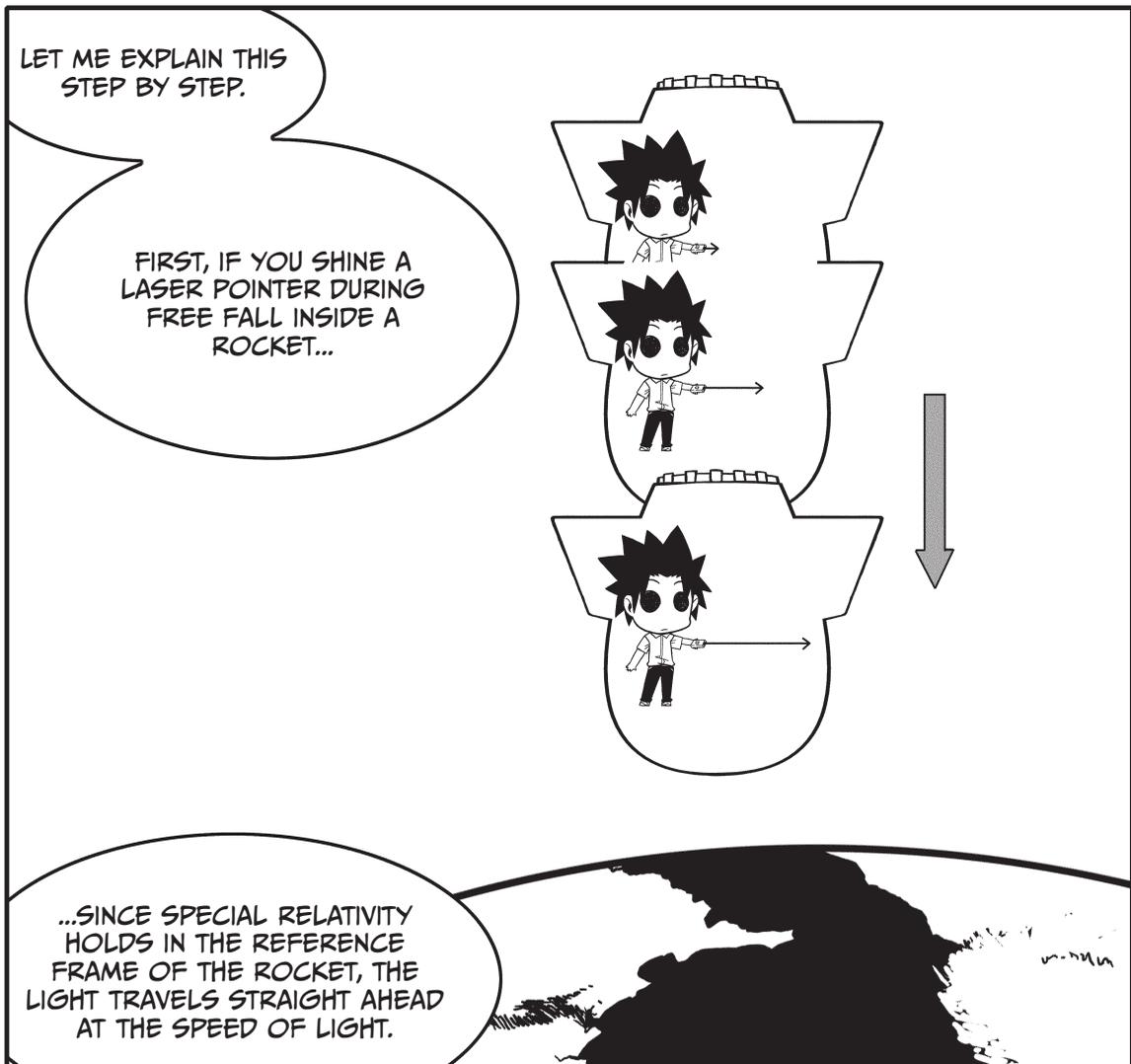
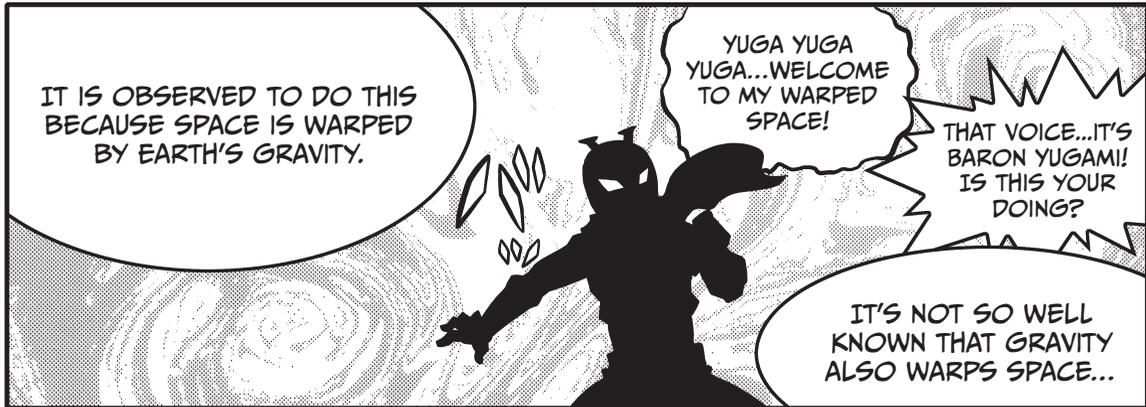
HEARING YOUR EXPLANATION,  
I THINK THAT THIS SURELY  
WOULD HAPPEN, BUT I'M  
NOT REALLY HAPPY WITH MY  
UNDERSTANDING OF THIS.

THE IMPORTANT  
POINT HERE...

IS THAT THE LIGHT, WHICH  
IS PROCEEDING STRAIGHT  
AHEAD INSIDE THE ROCKET,  
BENDS AS OBSERVED  
FROM THE GROUND.

MMHMMM?



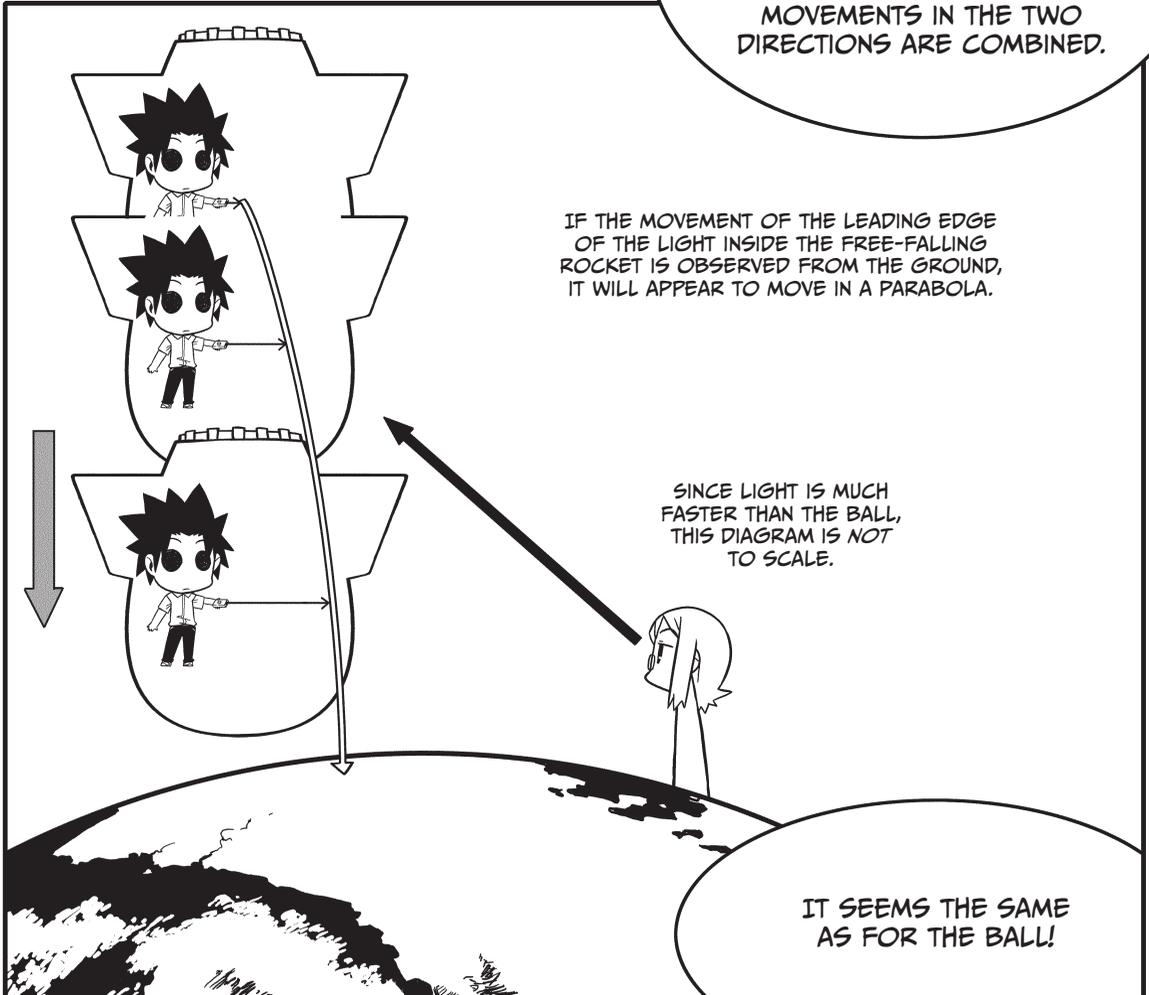


IF THE MOVEMENT OF THE LEADING EDGE OF THE LIGHT INSIDE THE FREE-FALLING ROCKET IS OBSERVED FROM THE GROUND, IT WILL APPEAR TO MOVE IN A PARABOLA.

THE LIGHT MOVES IN THE HORIZONTAL DIRECTION AT THE CONSTANT SPEED OF LIGHT ACCORDING TO SPECIAL RELATIVITY.

TOSS

SINCE IT FALLS IN THE VERTICAL DIRECTION ACCORDING TO CONSTANT GRAVITY, IT TRACES A PARABOLA WHEN THE MOVEMENTS IN THE TWO DIRECTIONS ARE COMBINED.



IF THE MOVEMENT OF THE LEADING EDGE OF THE LIGHT INSIDE THE FREE-FALLING ROCKET IS OBSERVED FROM THE GROUND, IT WILL APPEAR TO MOVE IN A PARABOLA.

SINCE LIGHT IS MUCH FASTER THAN THE BALL, THIS DIAGRAM IS NOT TO SCALE.

IT SEEMS THE SAME AS FOR THE BALL!

INSIDE THE FREE-FALLING ROCKET, THE LIGHT TRAVELS THE SHORTEST DISTANCE IN SPACE IN THE INERTIAL FRAME...OR, IN OTHER WORDS, IN A STRAIGHT LINE.

AND WHEN THE INTERIOR OF THE FREE-FALLING ROCKET IS OBSERVED FROM THE GROUND... THE LIGHT BENDS.

IT CERTAINLY APPEARS TO BE A ROUNDABOUT PATH THAT IS NOT TRAVELING ALONG THE SHORTEST DISTANCE.

DOESN'T THE WAY IN WHICH THE LIGHT TRAVELS APPEAR TO DIFFER WHEN IT IS OBSERVED FROM INSIDE THE ROCKET AND WHEN IT IS OBSERVED FROM THE GROUND?

YES, IT DOES, BUT...

GENERAL RELATIVITY ASSUMES THAT PHYSICAL PHENOMENA SUCH AS LIGHT TRAVELING ALONG THE SHORTEST DISTANCE IN SPACE-TIME...

SHOULD NOT DIFFER ACCORDING TO THE OBSERVATION POINT.

ISN'T THIS A CONTRADICTION?!

CALM DOWN.

SINCE SPACE-TIME IS FLAT IN AN INERTIAL FRAME, LIGHT IS OBSERVED TO PROCEED IN A STRAIGHT LINE...THIS IS WHAT HAPPENED, ISN'T IT?

UM...



WHEN THE PATH OF LIGHT INSIDE THE ACCELERATING ROCKET IS OBSERVED FROM THE GROUND, LIGHT MUST BE OBSERVED TO TAKE THE SHORTEST PATH THROUGH SPACE. HOWEVER, BECAUSE THE ROCKET IS ACCELERATING RELATIVE TO THE OBSERVERS ON EARTH, SPACE-TIME BECOMES BENT...

...SUCH THAT THE SHORTEST PATH FOR THE LIGHT APPEARS TO THEM TO BE A PARABOLA. THAT'S HOW EINSTEIN CONCEPTUALIZED IT.

WHAT?!

WHAT AN OUTRAGEOUS CONCEPT.

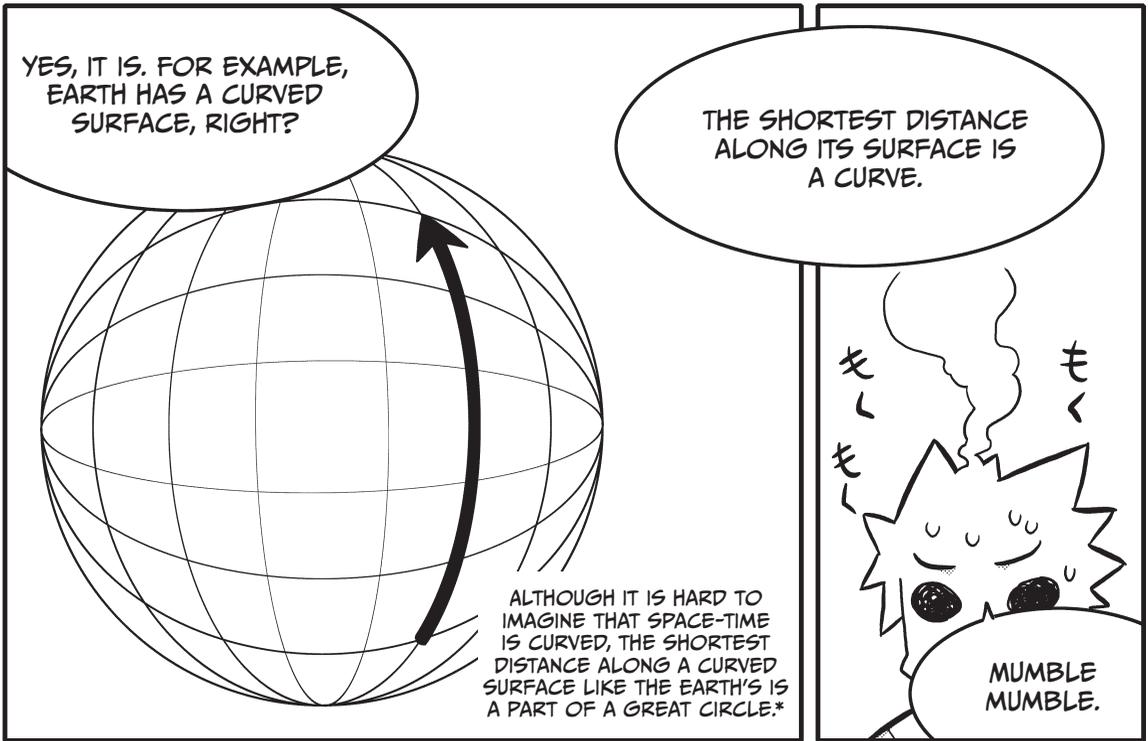


IT SURE IS.



AND EINSTEIN WONDERED WHETHER THAT CURVATURE OF SPACE-TIME WAS GRAVITY ITSELF.

IT'S REALLY HARD TO IMAGINE CURVED SPACE-TIME.

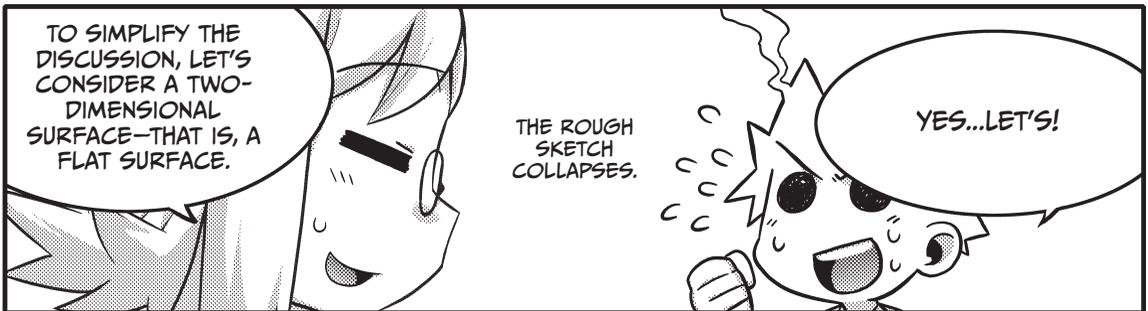


YES, IT IS. FOR EXAMPLE, EARTH HAS A CURVED SURFACE, RIGHT?

THE SHORTEST DISTANCE ALONG ITS SURFACE IS A CURVE.

ALTHOUGH IT IS HARD TO IMAGINE THAT SPACE-TIME IS CURVED, THE SHORTEST DISTANCE ALONG A CURVED SURFACE LIKE THE EARTH'S IS A PART OF A GREAT CIRCLE.\*

MUMBLE MUMBLE.

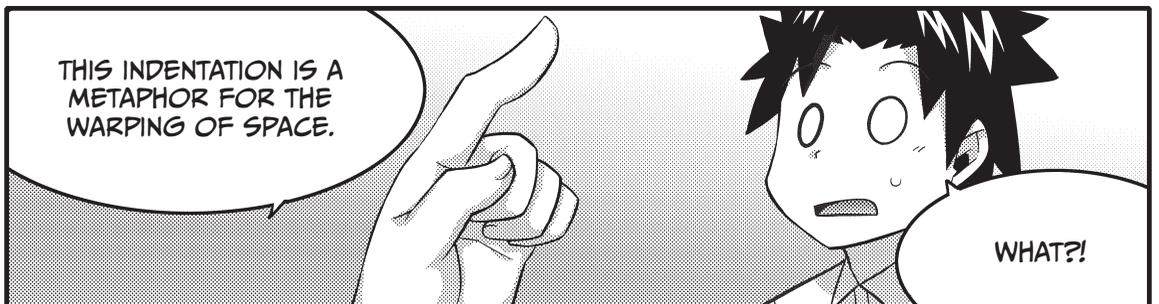
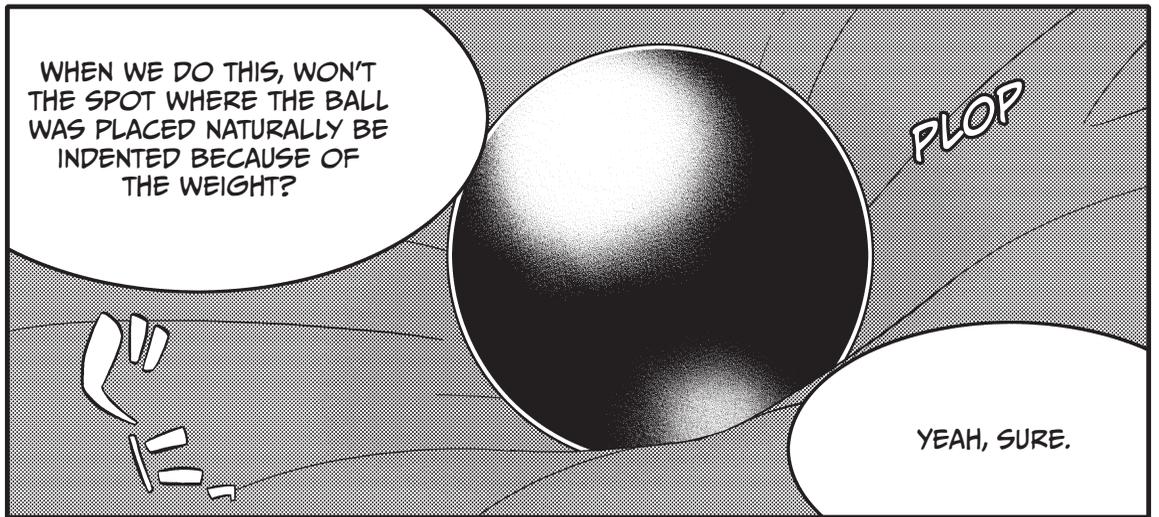
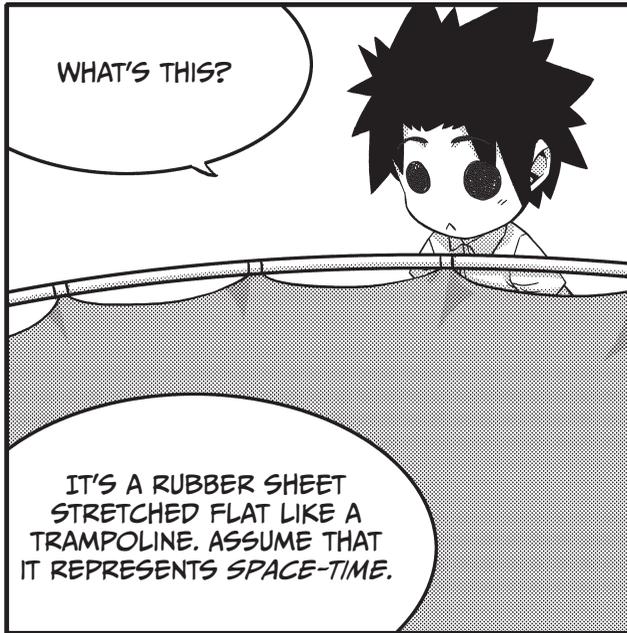


TO SIMPLIFY THE DISCUSSION, LET'S CONSIDER A TWO-DIMENSIONAL SURFACE—THAT IS, A FLAT SURFACE.

THE ROUGH SKETCH COLLAPSES.

YES...LET'S!

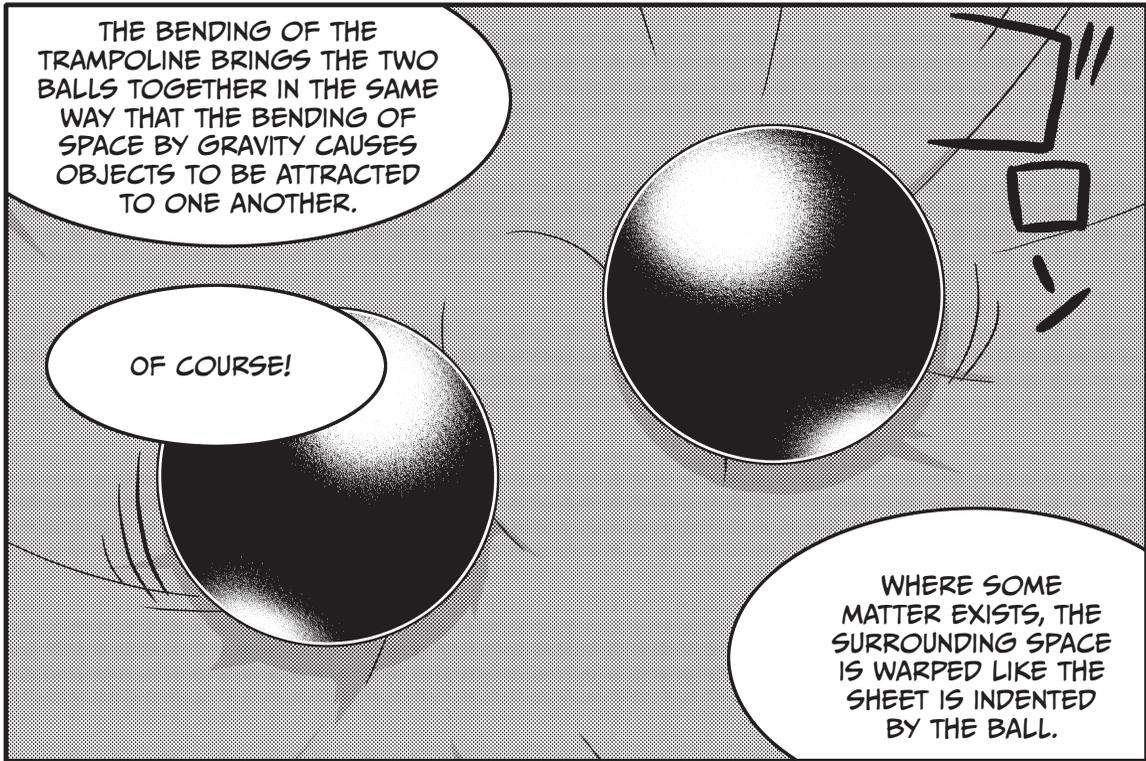
\* A GREAT CIRCLE IS ONE THAT EVENLY DIVIDES A SPHERE INTO TWO EQUAL HALVES. THE SHORTEST DISTANCE BETWEEN TWO POINTS ON A SPHERE IS ALWAYS A PORTION (OR A MINOR ARC) OF GREAT CIRCLE.





NOW LET'S ALSO PLACE ANOTHER BALL THERE.

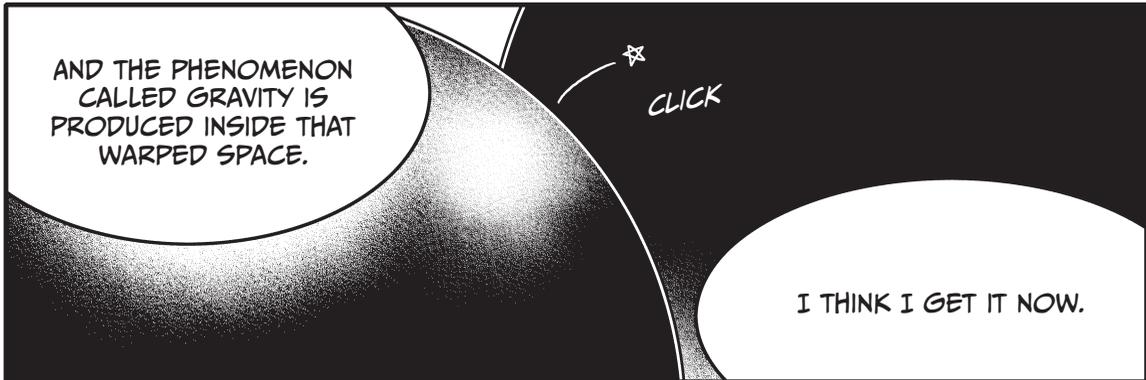
WHEN WE DO THAT, THE SIZE OF THE INDENTATION WILL INCREASE, AND THE TWO BALLS WILL APPROACH EACH OTHER.



THE BENDING OF THE TRAMPOLINE BRINGS THE TWO BALLS TOGETHER IN THE SAME WAY THAT THE BENDING OF SPACE BY GRAVITY CAUSES OBJECTS TO BE ATTRACTED TO ONE ANOTHER.

OF COURSE!

WHERE SOME MATTER EXISTS, THE SURROUNDING SPACE IS WARPED LIKE THE SHEET IS INDENTED BY THE BALL.

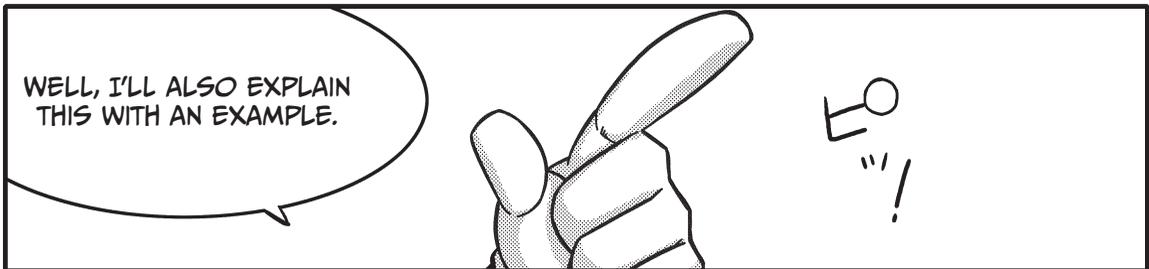
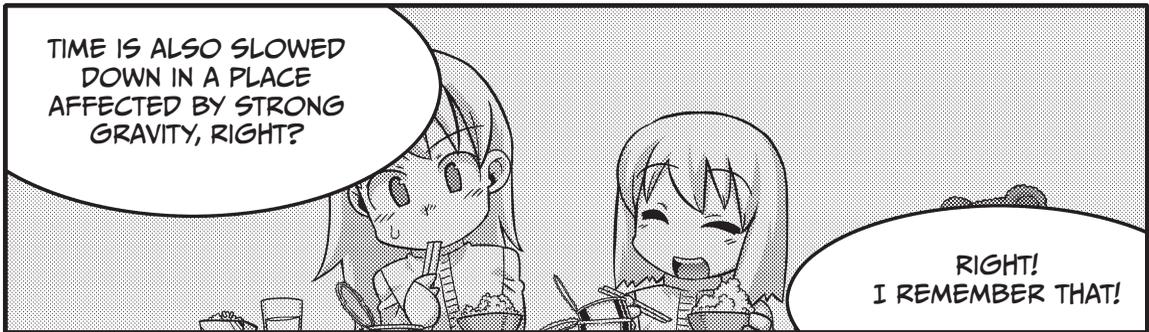


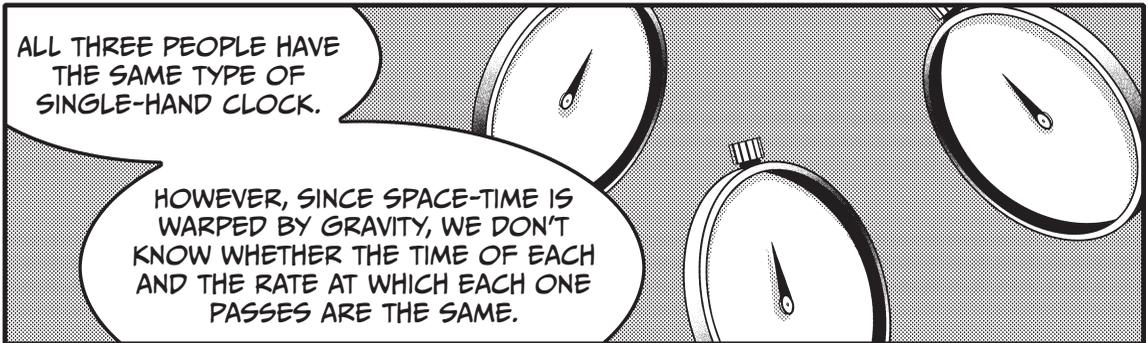
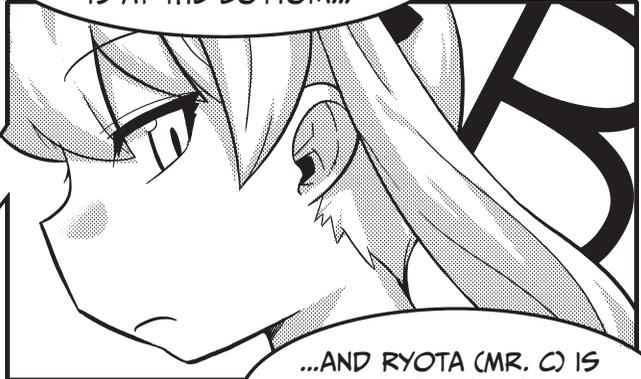
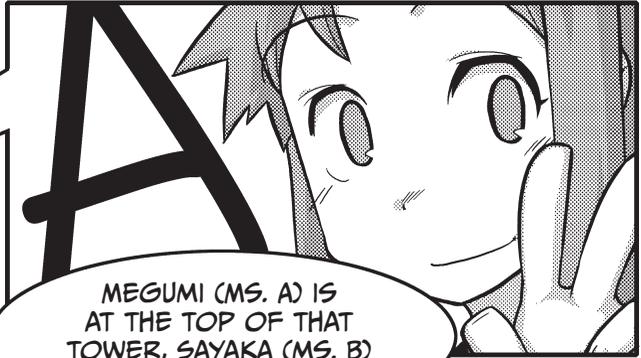
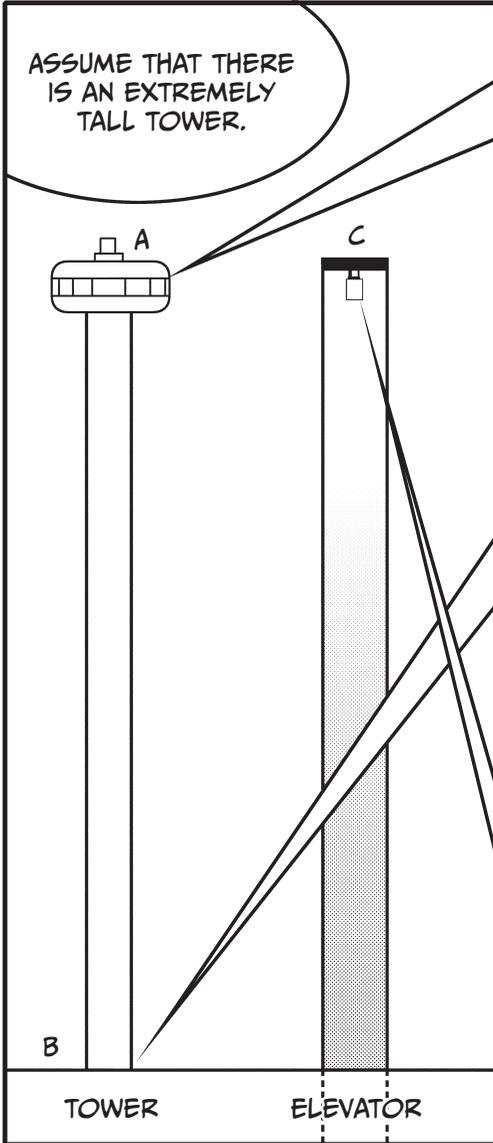
AND THE PHENOMENON CALLED GRAVITY IS PRODUCED INSIDE THAT WARPED SPACE.

I THINK I GET IT NOW.



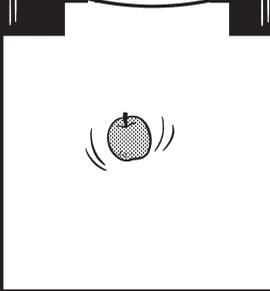
3. TIME IS SLOWED DOWN BY GRAVITY



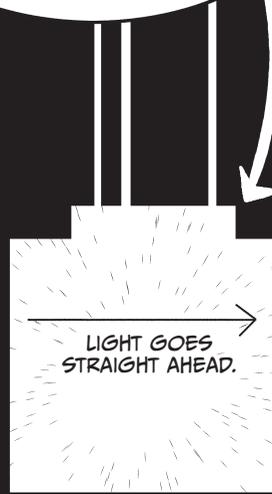


WE WILL VISUALLY COMPARE THE THREE PEOPLE'S CLOCKS WHEN THE ELEVATOR IS IN FREE FALL TO CHECK HOW GRAVITY AFFECTS THE RATE AT WHICH TIME PASSES. NOW, KEEP THE FOLLOWING POINTS IN MIND:

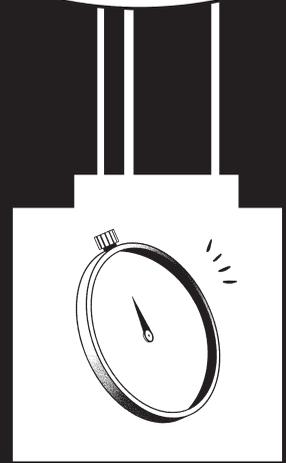
1. INSIDE THE FREE-FALLING ELEVATOR, THERE IS A STATE OF WEIGHTLESSNESS.



2. SPECIAL RELATIVITY HOLDS THERE.

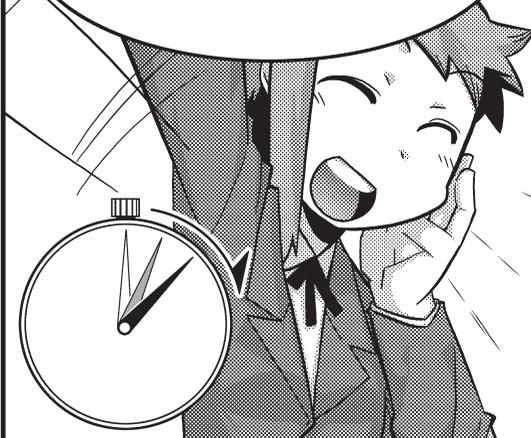


3. THEREFORE, THE CLOCK INSIDE THE ELEVATOR ADVANCES WITH CONSTANT TIME INTERVALS.



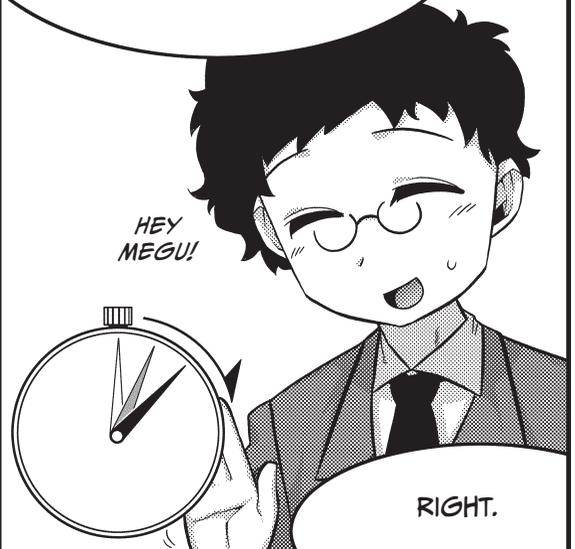
LET'S LOOK AT THIS METHODICALLY STEP-BY-STEP.

SINCE MS. A AND MR. C ARE AT THE SAME HEIGHT AT FIRST, THEY ARE SUBJECT TO THE SAME GRAVITY.

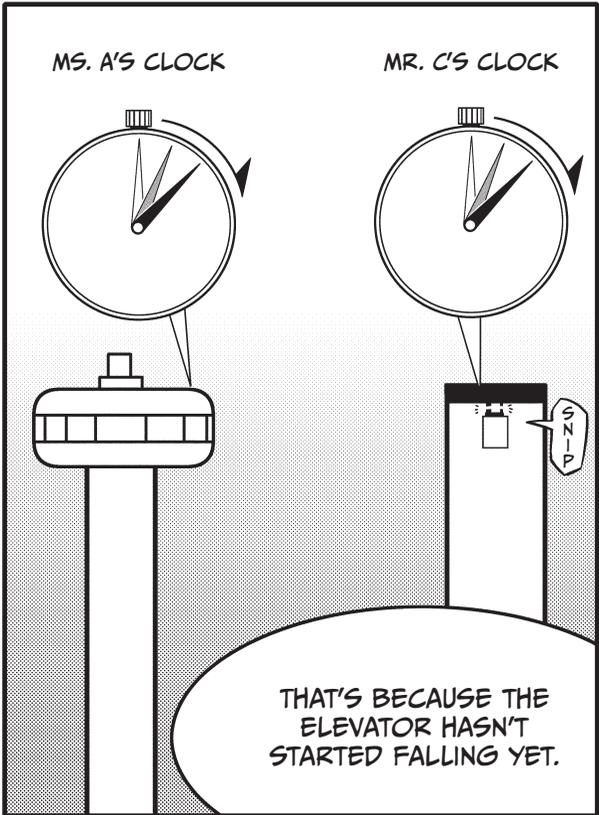
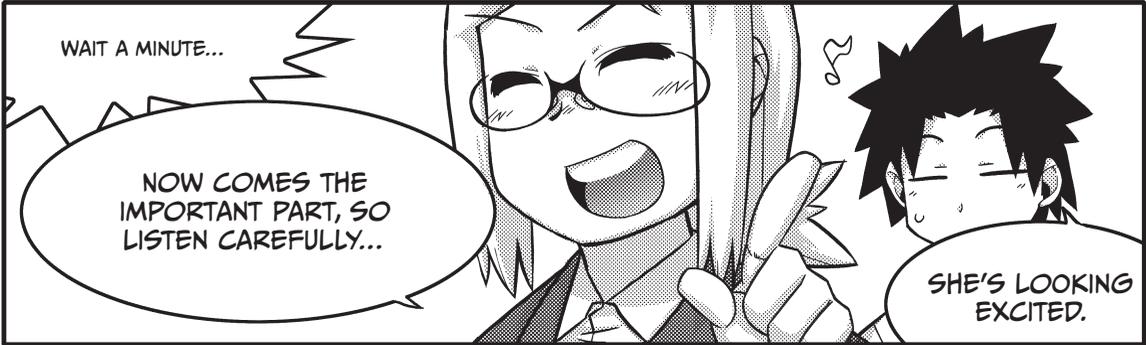


THEREFORE, THE TIME AND RATE AT WHICH TIME PASSES MATCH FOR MS. A AND MR. C.

HEY MEGU!



RIGHT.



THE ELEVATOR IS PULLED BY GRAVITY, AND ITS VELOCITY INCREASES.



AND THE ELEVATOR...



...PASSES ALONGSIDE MS. B AT A CERTAIN VELOCITY.

WAIT, WHAT ABOUT MR. C?! IS HE SAFE?!

YES, IF WE DREW A PICTURE, IT WOULD LOOK LIKE THIS.



MS. B



MR. C

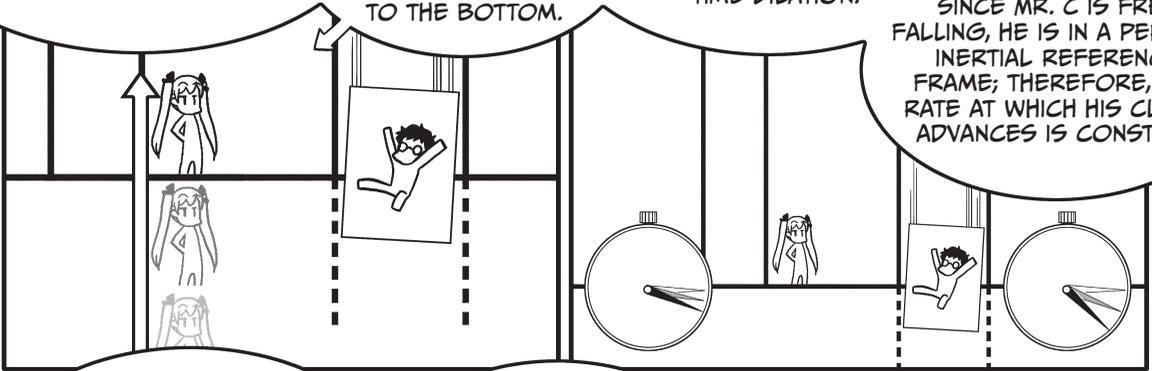
ANSWER ME!

IF MS. B WERE VIEWED AT THAT TIME BY MR. C FROM INSIDE THE ELEVATOR, HE WOULD OBSERVE HER TO BE MOVING FROM THE BOTTOM TOWARDS THE TOP...

WHICH IS THE REVERSE OF HIS OWN MOTION OF FALLING FROM THE TOP OF THE TOWER TO THE BOTTOM.

ACCORDING TO SPECIAL RELATIVITY, AT THE INSTANT THAT MR. C PASSES MS. B, MR. C OBSERVES MS. B'S TIME SLOW DOWN DUE TO TIME DILATION.

SINCE MR. C IS FREE FALLING, HE IS IN A PERFECT INERTIAL REFERENCE FRAME; THEREFORE, THE RATE AT WHICH HIS CLOCK ADVANCES IS CONSTANT.



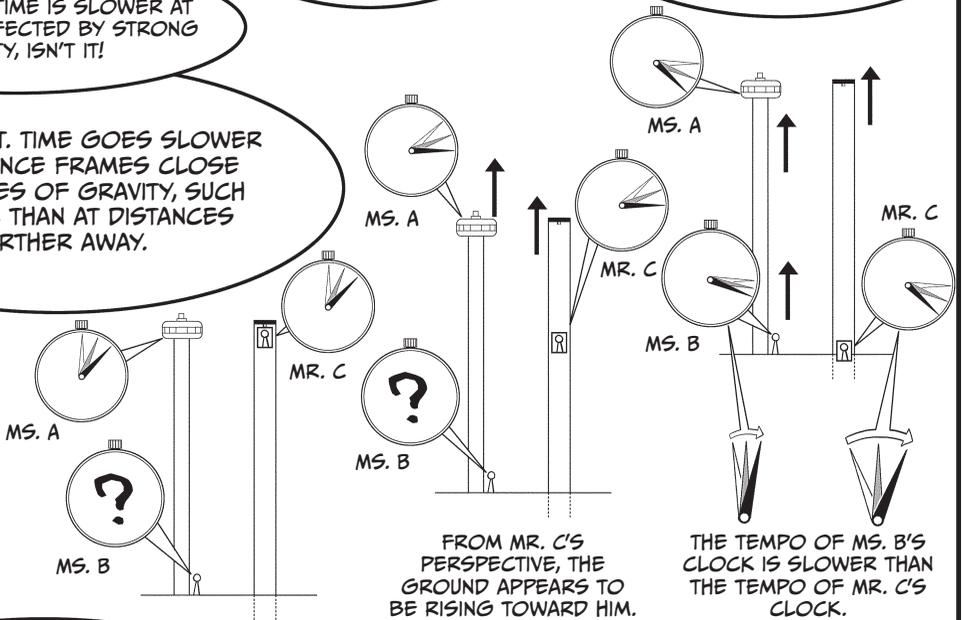
LET'S ORGANIZE ALL THIS USING MR. C AS THE REFERENCE. WHEN THE ELEVATOR STARTED TO FALL, THE TEMPOS OF MS. A'S AND MR. C'S CLOCKS WERE THE SAME.

WHEN MR. C FINALLY HITS THE GROUND, WE TAKE A LOOK AT ALL THREE OF THE CLOCKS. MS. B'S CLOCK HAS GONE SLOWER THAN MR. C'S BECAUSE OF TIME DILATION.

AND BECAUSE MR. C'S AND MS. A'S CLOCKS STARTED AT THE SAME TEMPO, MS. B'S CLOCK HAS GONE SLOWER THAN MS. A'S.

THE FLOW OF TIME IS SLOWER AT A LOCATION AFFECTED BY STRONG GRAVITY, ISN'T IT!

THAT'S RIGHT. TIME GOES SLOWER IN REFERENCE FRAMES CLOSE TO SOURCES OF GRAVITY, SUCH AS EARTH, THAN AT DISTANCES FURTHER AWAY.



TO SAY IT MORE PRECISELY, THE LOWER THE GRAVITATIONAL POTENTIAL IS, THE MORE SLOWLY TIME PASSES.

#### 4. RELATIVITY AND THE UNIVERSE

TIME AND SPACE OR,  
IN OTHER WORDS,  
SPACE-TIME,

WHICH HAD BEEN  
CONSIDERED  
TO SIMPLY BE A  
RECEPTACLE FOR  
MATTER...

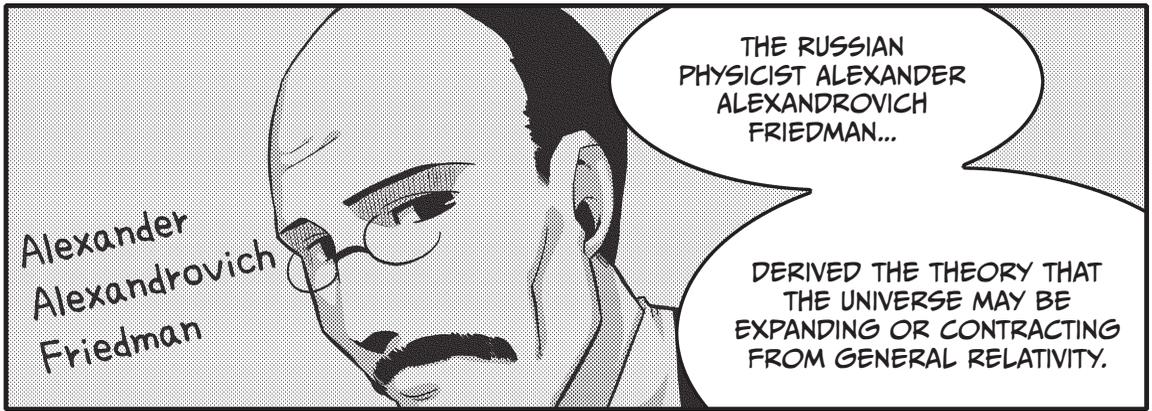
...MUST BE CONSIDERED  
TOGETHER WITH MATTER.  
MUTUALLY INTERACTIVE  
RELATIONSHIPS AMONG THEM  
HAVE BEEN REVEALED.

EVEN IF YOU UNDERSTAND  
THE THEORY, IT STILL SEEMS  
STRANGE.

BUT THIS CONCEPT  
HAS A SIGNIFICANT  
EFFECT ON HOW WE  
PERCEIVE THE SPACE  
SURROUNDING US.

IN FACT, MODERN  
COSMOLOGY IS  
ENTIRELY DEPENDENT  
ON GENERAL  
RELATIVITY.

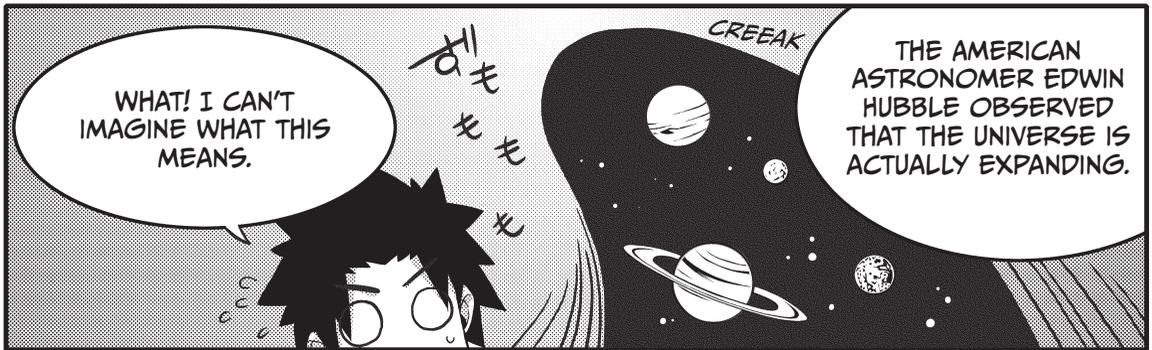
HUH!



Alexander Alexandrovich Friedman

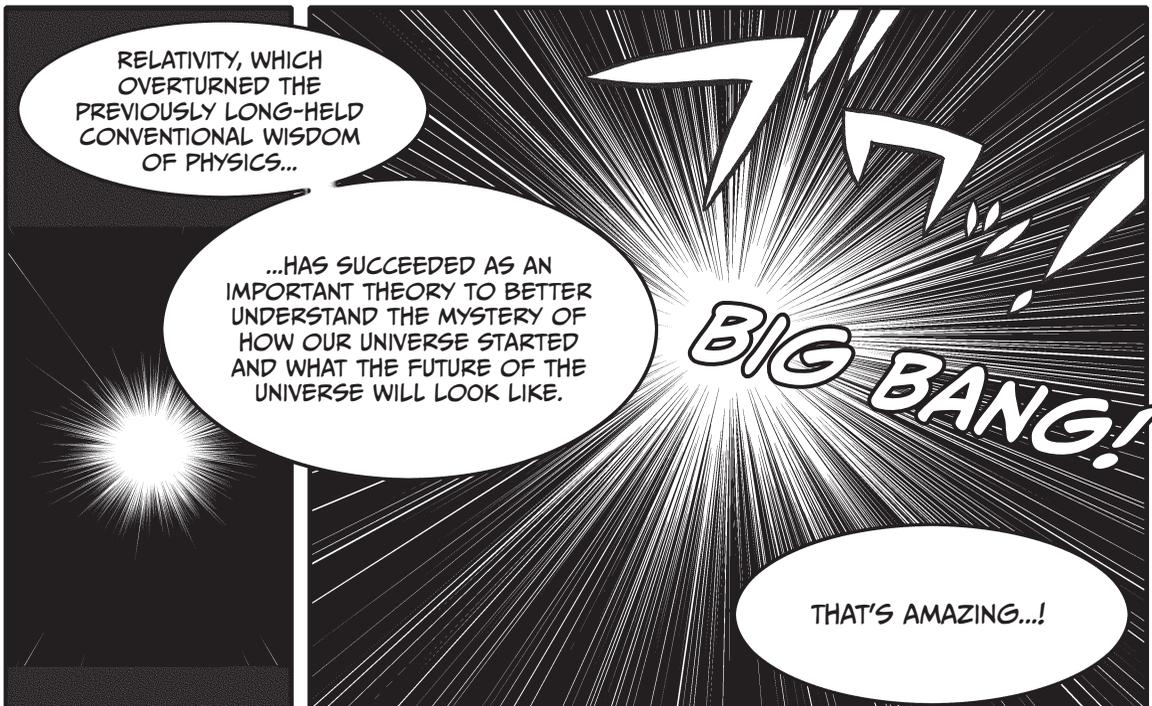
THE RUSSIAN PHYSICIST ALEXANDER ALEXANDROVICH FRIEDMAN...

DERIVED THE THEORY THAT THE UNIVERSE MAY BE EXPANDING OR CONTRACTING FROM GENERAL RELATIVITY.



WHAT! I CAN'T IMAGINE WHAT THIS MEANS.

THE AMERICAN ASTRONOMER EDWIN HUBBLE OBSERVED THAT THE UNIVERSE IS ACTUALLY EXPANDING.



RELATIVITY, WHICH OVERTURNED THE PREVIOUSLY LONG-HELD CONVENTIONAL WISDOM OF PHYSICS...

...HAS SUCCEEDED AS AN IMPORTANT THEORY TO BETTER UNDERSTAND THE MYSTERY OF HOW OUR UNIVERSE STARTED AND WHAT THE FUTURE OF THE UNIVERSE WILL LOOK LIKE.

**BIG BANG!**

THAT'S AMAZING...!

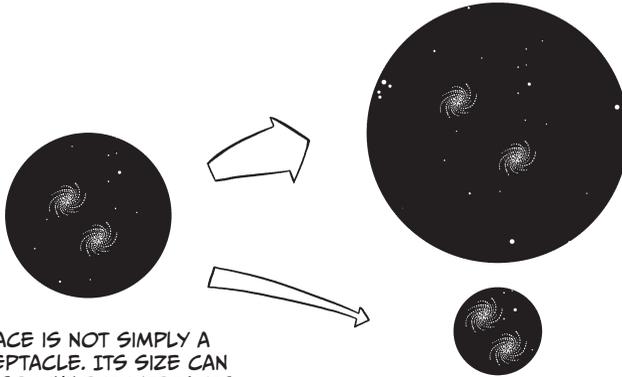
ALTHOUGH THERE ARE VARIOUS MODELS OF THE UNIVERSE, THE THEORY DERIVED FROM GENERAL RELATIVITY...

...WHICH SAYS THAT "THE UNIVERSE MAY BE EXPANDING"...

STRETCH

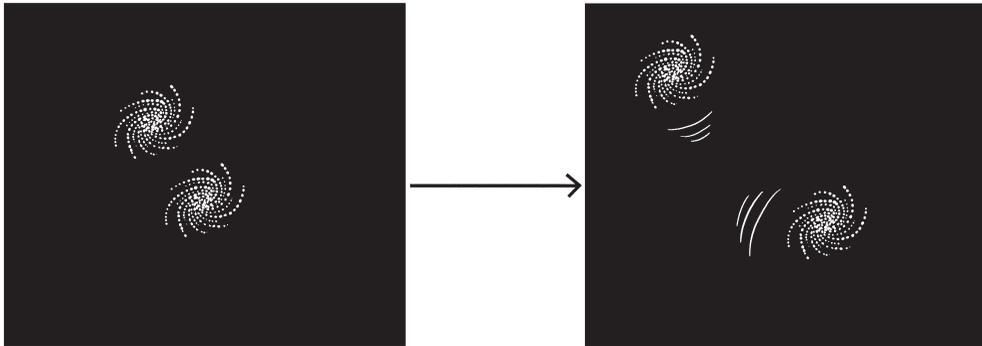
HAS BECOME THE PREDOMINANT MODEL OF THE UNIVERSE.

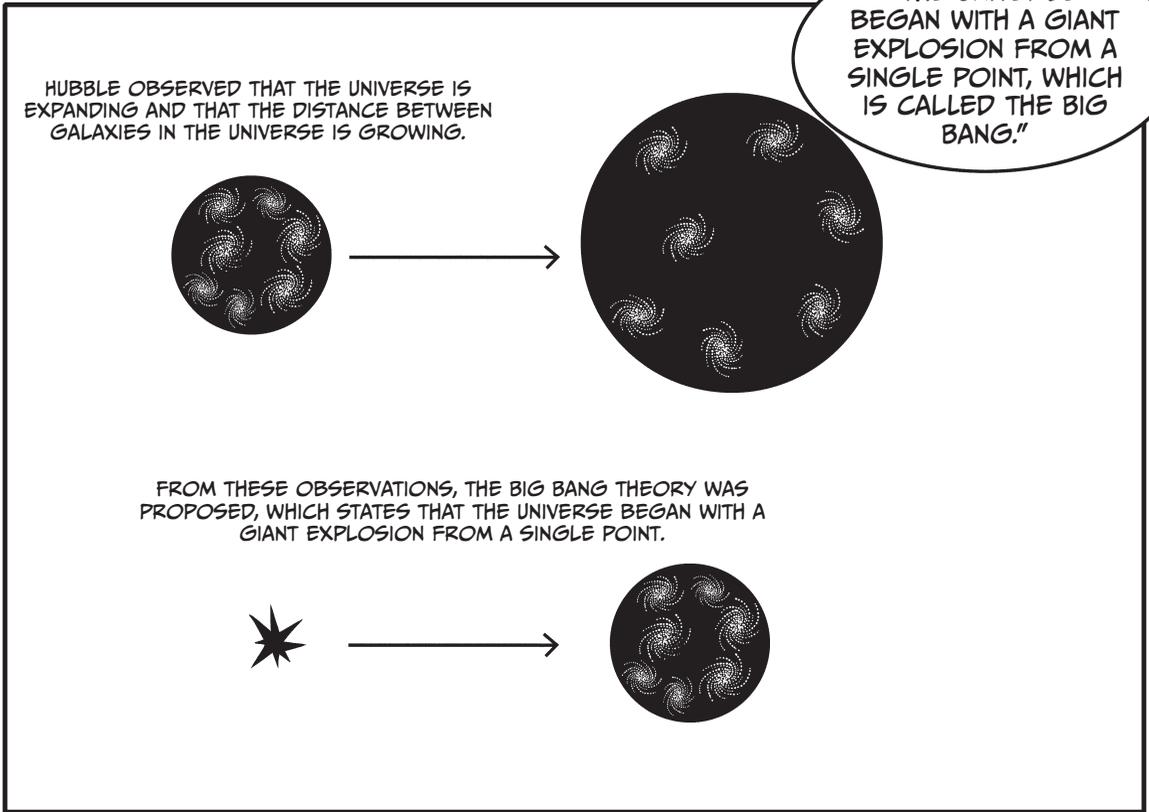
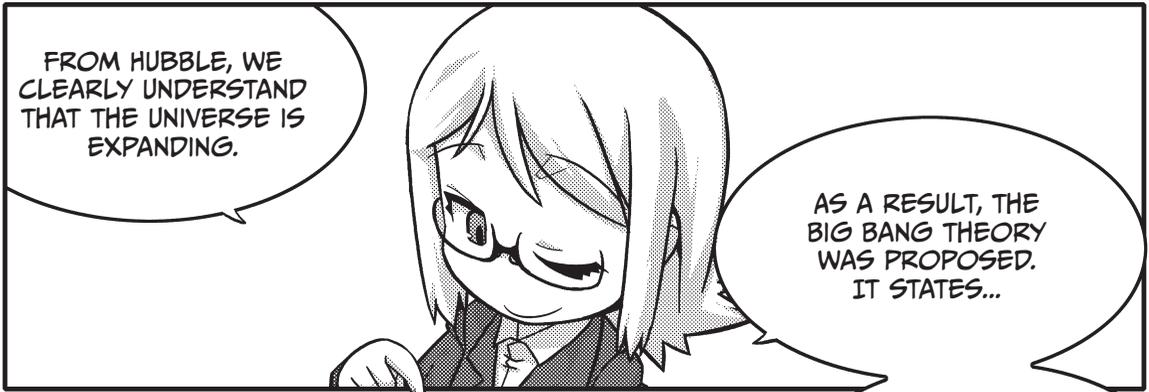
IT IS KNOWN THAT THE UNIVERSE MAY BE EXPANDING OR CONTRACTING AS SPACE-TIME.

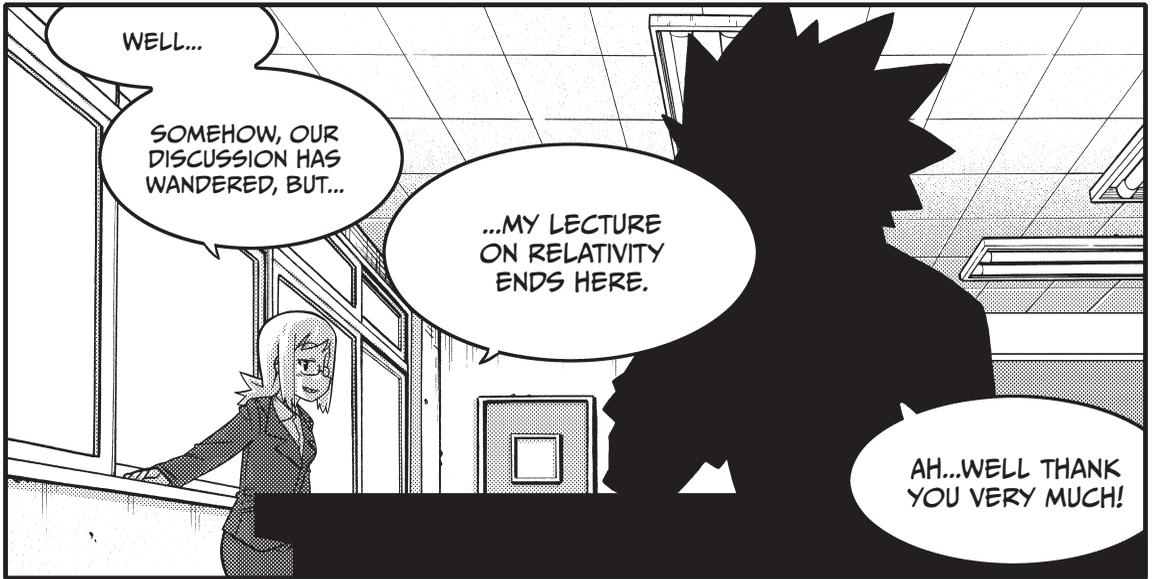


SPACE IS NOT SIMPLY A RECEPTACLE. ITS SIZE CAN CHANGE WHILE IT INTERACTS WITH MATTER.

ALTHOUGH NEWTONIAN MECHANICS CONSIDERED MATTER TO BE SPREAD OUT IN "SPACE," WHICH HAD AN INFINITE EXpanse, IT DID NOT ASSUME THAT SPACE ITSELF WAS SPREADING OUT.







WELL...

SOMEHOW, OUR DISCUSSION HAS WANDERED, BUT...

...MY LECTURE ON RELATIVITY ENDS HERE.

AH...WELL THANK YOU VERY MUCH!



LATER YOU'LL GET THE BETTER OF THE HEADMASTER BECAUSE OF YOUR EFFORT! FOR MY SAKE!

I HOPE SO...

BUT...



DID YOU KNOW THERE IS A FIREWORKS DISPLAY TONIGHT?

I WAS TALKING ABOUT IT THIS MORNING...

HOW ABOUT IT? SHALL WE GO TOGETHER?

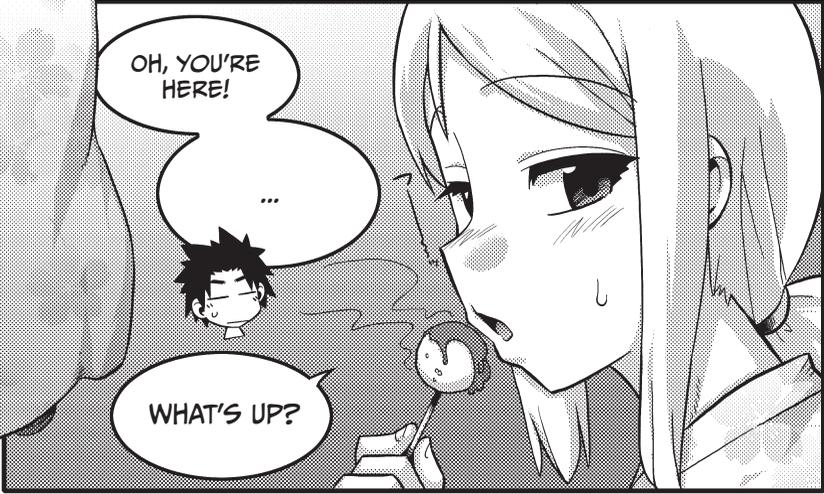
HUH?!



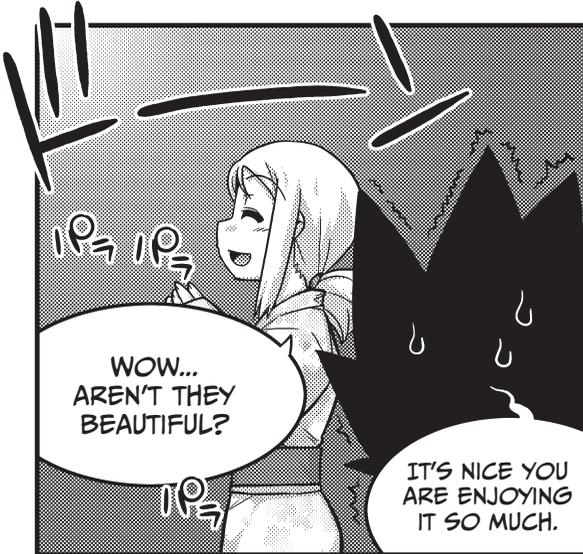
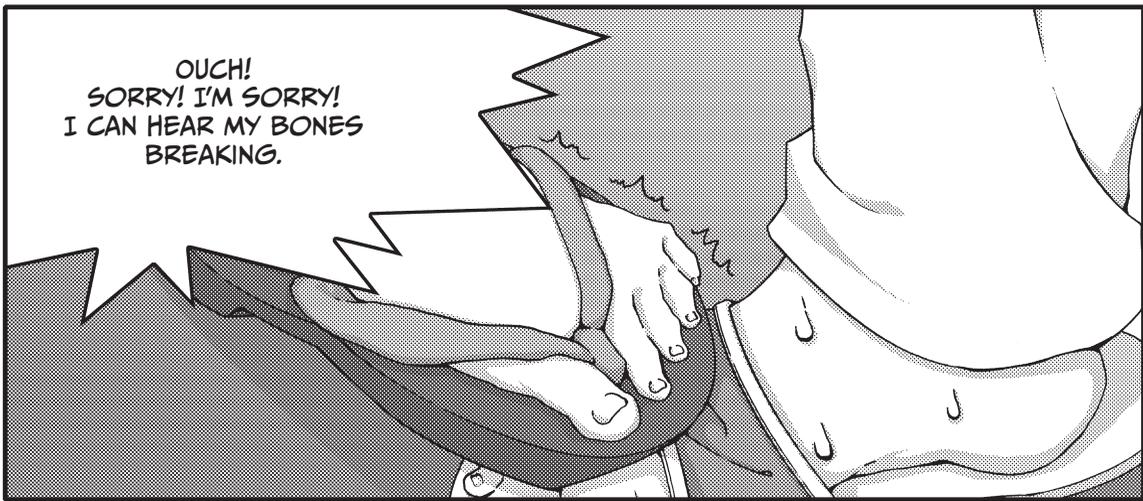
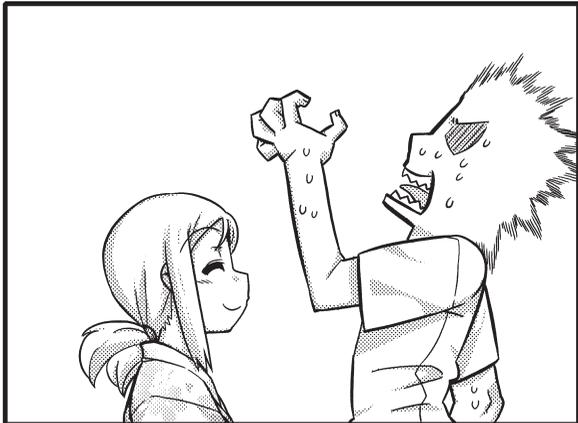
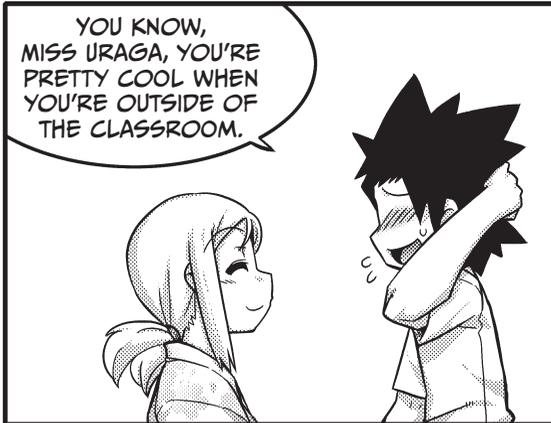
YOU CAN TREAT ME TO GRILLED OCTOPUS.

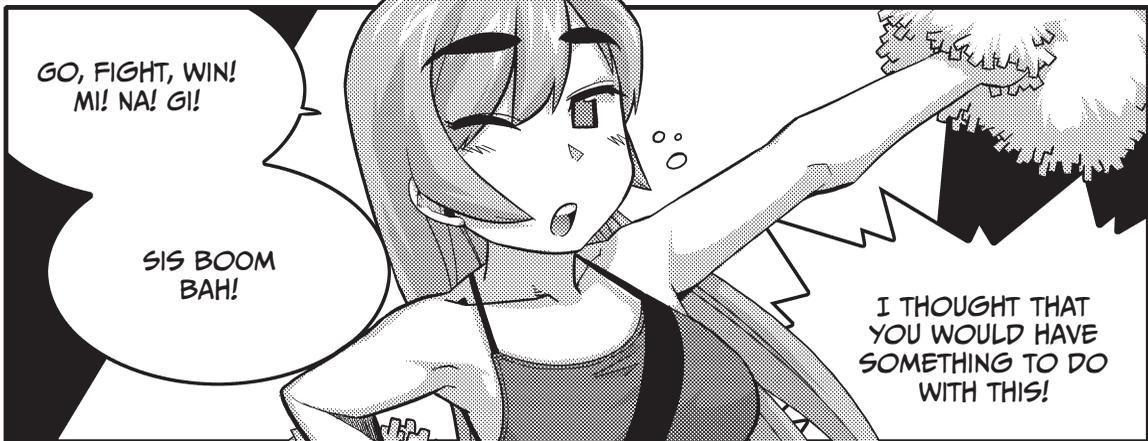
OOOOH...YOU SURE ARE AN UNPLEASANT ADULT!

FINE, I'LL GO.



02







CAN WE GET A SPOTLIGHT ON THE MAN OF HONOR PLEASE?

KA-CHUNK

WOW...

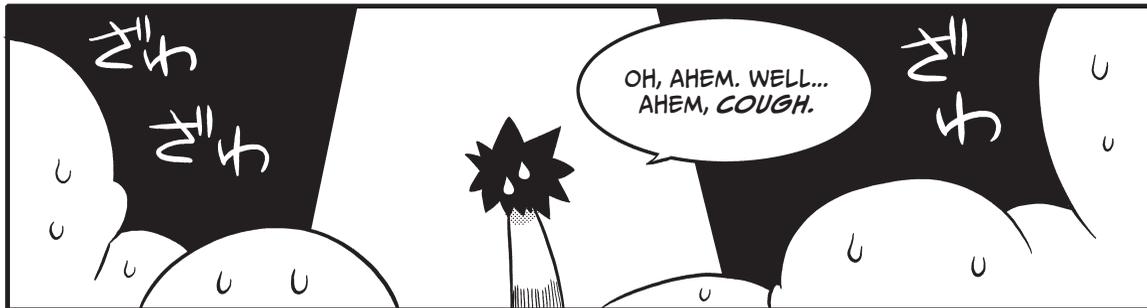


MISS URAGA, HELP ME!

EVERYONE IS CHEERING AND YOU'RE BUYING YAKISOBA!

HERE YOU GO, HONEY. ON THE HOUSE!

YOU'RE TOO GENEROUS.



OH, AHEM. WELL... AHEM, COUGH.



HEY! ARE YOU PLANNING ON JOINING THE CHEERLEADERS NOW?

HUH? IS THAT WHAT THIS IS?



NO. THESE ARE JUST MY REGULAR CLOTHES.

I DON'T KNOW WHAT THIS IS ABOUT!

# THE SLOWING OF TIME IN GENERAL RELATIVITY

Let's use some equations to look at the "slowing of time" in general relativity based on the explanation in the manga.

As in the manga, we assume that Ms. A is at the top of a tall tower, Ms. B is at the bottom of the tower, and Mr. C is inside the elevator next to the tower, as shown in Figure 4-1.

We also assume that each of the three people has the same clock. However, since space-time is warped by gravity, we don't know whether the time of each and the rate at which each one passes (tempo) are the same.

Therefore, we will check the rate at which time passes due to gravity under the following three conditions:

1. Inside the free-falling elevator, there is a state of weightlessness.
2. Since special relativity holds there, the clock inside the elevator advances with constant time intervals.
3. Ms. A's clock at the top of the tower and Ms. B's clock at the bottom of the tower each advance with different constant time intervals.

In addition, we will use the following procedure to check the rate at which time passes due to gravity.

1. Align the rates at which time passes for Mr. C's clock inside the elevator and Ms. A's clock at the beginning of the descent.
2. Compare the rates at which time passes for Mr. C's clock inside the elevator and Ms. B's clock at the end of the descent.

At first, since Ms. A and Mr. C are at the same height, they are affected by the same gravity.

Let  $z$  denote the height direction at that location, and let  $\phi_1$  denote the gravitational potential. The gravitational potential is the quotient of the potential energy divided by the mass of an object. For example, the potential energy of gravity near the surface of the Earth is  $mgh$ , and the gravitational potential is  $gh$ .

Therefore, we will align Ms. A's and Mr. C's times and the rates at which time passes.

Let  $\Delta\tau_1$  denote the time that passes at Ms. A's location, and let  $\Delta\tau_2$  denote the time that passes at Ms. B's location.

Now let's assume that the cable that is holding the elevator is cut and the elevator starts to free-fall. Since the falling velocity immediately after the cable is cut (the velocity at which Ms. A is flying upward when viewed from Mr. C's perspective) is  $v = 0$ , the tempos of Ms. A's and Mr. C's clocks are the same.

$$\text{①} \quad \Delta\tau_1 = \Delta\tau_3$$

The elevator is pulled by gravity, and its velocity gradually increases. The elevator passes alongside Ms. B at a certain velocity ( $v$ ).

If Ms. B were viewed at that time by Mr. C inside the elevator, he would observe her to be moving upward toward himself, which is the reverse of his own motion (falling from the top of the tower toward the bottom) viewed from his surroundings (see Figure 4-2).

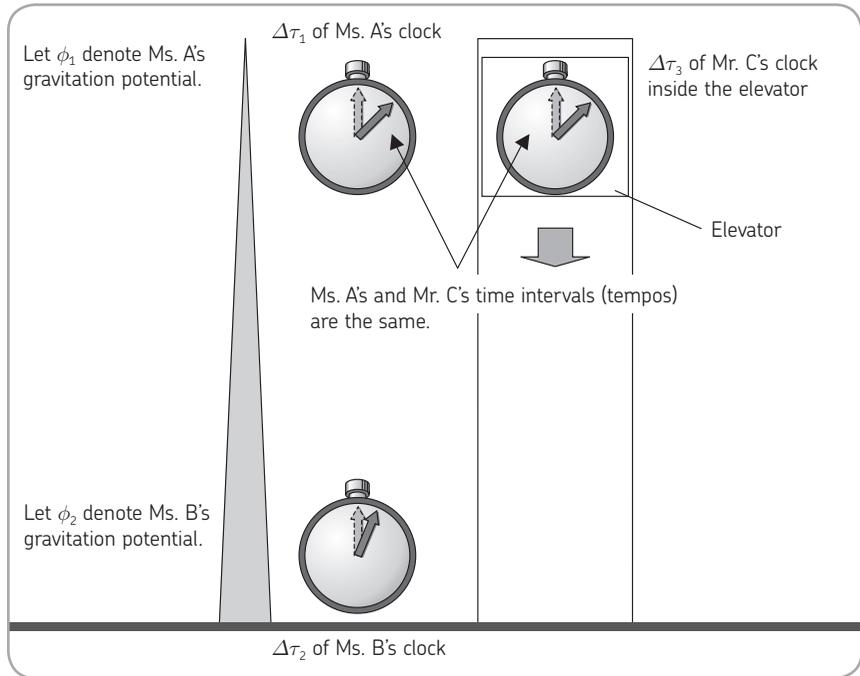


Figure 4-1: Aligning the rates at which time passes for Mr. C's clock inside the elevator and Ms. A's clock at the beginning of the descent

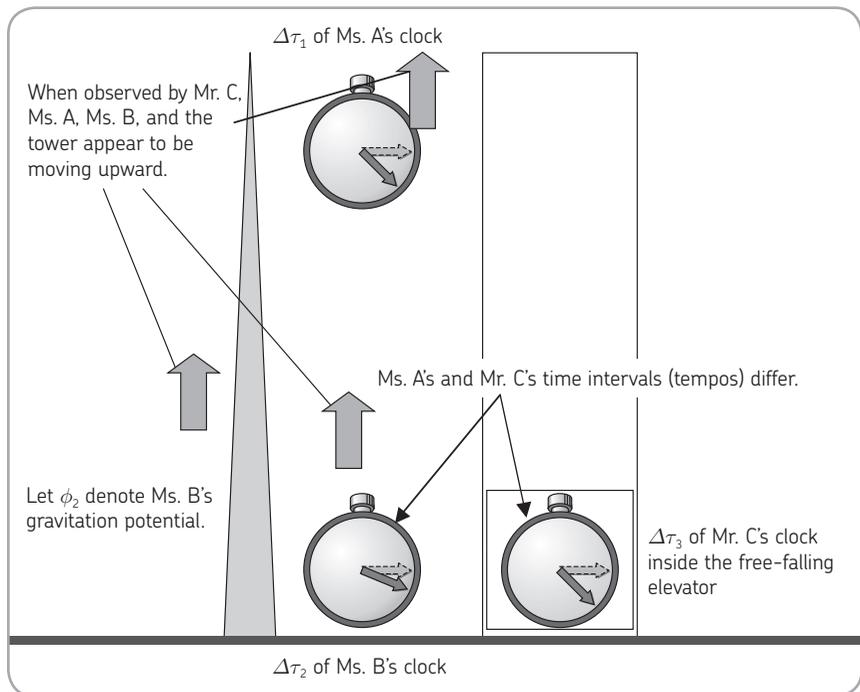


Figure 4-2: Comparing the rates at which time passes for the clocks of Mr. C inside the elevator and Ms. B at the end of the descent

According to special relativity, at the instant that Mr. C passed Ms. B, we have

$$\textcircled{2} \quad \Delta\tau_2 = \Delta\tau_3 \sqrt{1 - \left(\frac{v}{c}\right)^2},$$

and from equations  $\textcircled{1}$  and  $\textcircled{2}$ , we eliminate  $\Delta\tau_3$  to obtain

$$\textcircled{3} \quad \frac{\Delta\tau_2}{\Delta\tau_1} = \sqrt{1 - \left(\frac{v}{c}\right)^2} < 1.$$

This shows that the tempo of Ms. B's clock becomes slower than the tempo of Mr. C's clock. The rate at which time passes for Ms. B's clock, where the gravitational potential is low ( $\phi_2$ , close to the source of gravity), is slower than the rate at which time passes for Ms. A's clock, where the gravitational potential is high ( $\phi_1$ , far from the source of gravity).

In other words, the lower the gravitational potential is, the slower time will pass.

Let's assume that the velocity  $v$  is low and that we can use Newtonian mechanics (if we let  $x = \frac{v}{c}$ , then  $x \ll 1$ ).

Therefore, when  $\phi_1$  denotes the gravitational potential at Ms. A's location and  $\phi_2$  denotes the gravitational potential at Ms. B's location, we have  $\phi_1 > \phi_2$ .

Due to the conservation of energy, at the instant before the elevator hits the ground, all of its potential energy has been converted to kinetic energy, and its velocity  $v$  is given by this expression:

$$(\phi_1 - \phi_2)m = \frac{1}{2}mv^2$$

We obtain the following:

$$\textcircled{4} \quad \phi_1 - \phi_2 = \frac{1}{2}v^2$$

When  $x \ll 1$ , we can use the following approximation formula:

$$(1+x)^\alpha \approx 1 + \alpha x$$

Since  $x \ll 1$  because  $x = \frac{v}{c}$ , we have the following:

$$\sqrt{1 - \left(\frac{v}{c}\right)^2} = (1 - x^2)^{\frac{1}{2}} \approx 1 - \frac{1}{2}x^2 = 1 - \frac{1}{2}\left(\frac{v}{c}\right)^2$$

If we use this together with equation ③, we obtain the following equation:

$$\textcircled{5} \quad \frac{\Delta\tau_2}{\Delta\tau_1} = \sqrt{1 - \left(\frac{v}{c}\right)^2} \approx 1 - \frac{1}{2}\left(\frac{v}{c}\right)^2$$

If we substitute  $\frac{1}{2}v^2 = \phi_1 - \phi_2$  from equation ④ into equation ⑤, we obtain the following equation:

$$\textcircled{6} \quad \frac{\Delta\tau_2}{\Delta\tau_1} \approx 1 - \frac{1}{2}\left(\frac{v}{c}\right)^2 = 1 - \frac{\phi_1 - \phi_2}{c^2}$$

Also, if we rearrange the above equation slightly, since  $\frac{\phi_1 - \phi_2}{c^2} \approx 1 - \frac{\Delta\tau_2}{\Delta\tau_1} = \frac{\Delta\tau_1 - \Delta\tau_2}{\Delta\tau_1}$  holds, we have the following:

$$\textcircled{7} \quad \frac{\Delta\tau_1 - \Delta\tau_2}{\Delta\tau_1} \approx \frac{\phi_1 - \phi_2}{c^2}$$

In other words, gravitational potential is related to the slowing of time (time dilation), as shown in equation ⑦.

As shown in Figure 4-3, the difference in gravitational potential  $\phi_1 - \phi_2$  between the ground and an object at a height  $h$  is given by the gravitational potential  $gh$ .

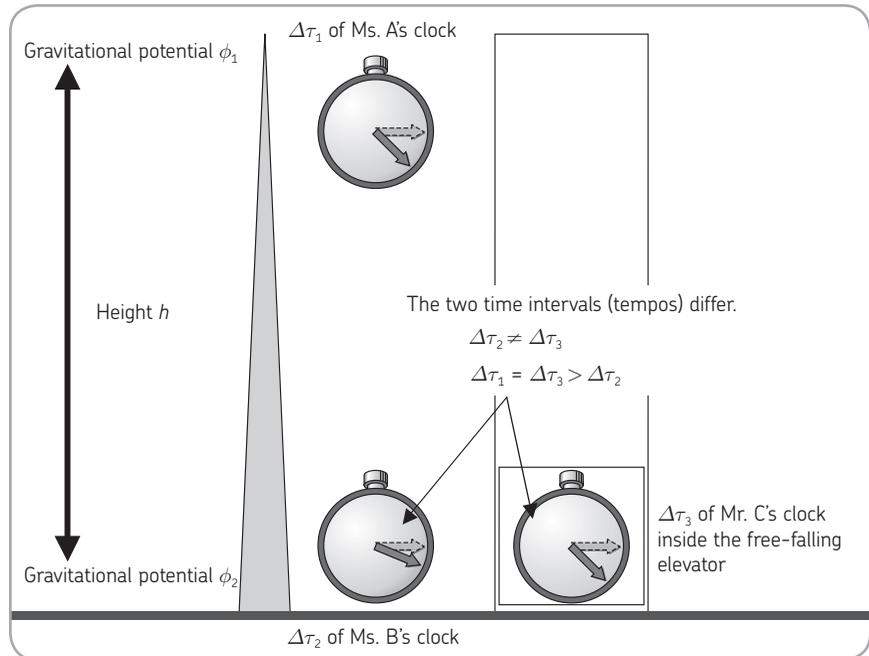


Figure 4-3: State of relatively weak gravity on the ground

If we let  $\phi_2 = 0$ ,  $h$  denote the height up to  $\phi_1$ , and  $g$  denote the gravitational acceleration near the ground and then substitute  $\phi_1 = gh$  and  $\phi_2 = 0$  into equation 7, we obtain the following:

$$\frac{\Delta\tau_1 - \Delta\tau_2}{\Delta\tau_1} \approx \frac{\phi_1 - \phi_2}{c^2} = \frac{gh - 0}{c^2} = \frac{gh}{c^2}$$

The clock at the higher altitude measures a time  $\Delta\tau_1$  that is ahead of the clock below, which reads  $\Delta\tau_2$ . We know this to be true, because in the equation above,  $gh/c^2$  and  $\Delta\tau_1$  are both greater than zero, and therefore  $\Delta\tau_1 - \Delta\tau_2 > 0$ ; hence,  $\Delta\tau_1 > \Delta\tau_2$ .

## THE TRUE NATURE OF GRAVITY IN GENERAL RELATIVITY

Space-time surrounding the existence of matter is warped, as explained in the manga. It is apparent that this warping of space-time has the same effect as gravity in attracting surrounding matter.

Einstein unified these effects in a set of equations called the *Einstein field equations*. The Einstein field equations showed that time and space (space-time), which were previously thought to exist as a framework for measuring the motion of matter, were fundamentally connected with matter itself.

## PHENOMENA DISCOVERED FROM GENERAL RELATIVITY

This section introduces the following phenomena, which were discovered from general relativity:

- Gravitational lensing
- Anomalous perihelion precession of Mercury
- Black holes

### BENDING OF LIGHT (GRAVITATIONAL LENSING) NEAR A LARGE MASS (SUCH AS THE SUN)

*Gravitational lensing* is the phenomenon that when light passes in the vicinity of the Sun, the path of that light bends.

Space is bent in the vicinity of the Sun because of the large mass of the Sun, as shown in Figure 4-4. Since light advances along that curvature, the light from a distant star bends, and the direction of the star is observed with a slight shift. This effect was verified during a total solar eclipse. It is noted as the first proof discovered of general relativity.

Also, when light is coming from a distant galaxy, as shown in Figure 4-5, if a massive object (such as a galaxy) lies at an intermediate point, it will bend the light from the distant galaxy as though there were a condensing lens at that intermediate point. This bend may make the distant galaxy seem distorted. Many instances of this effect have been observed. This is another instance of gravitational lensing.

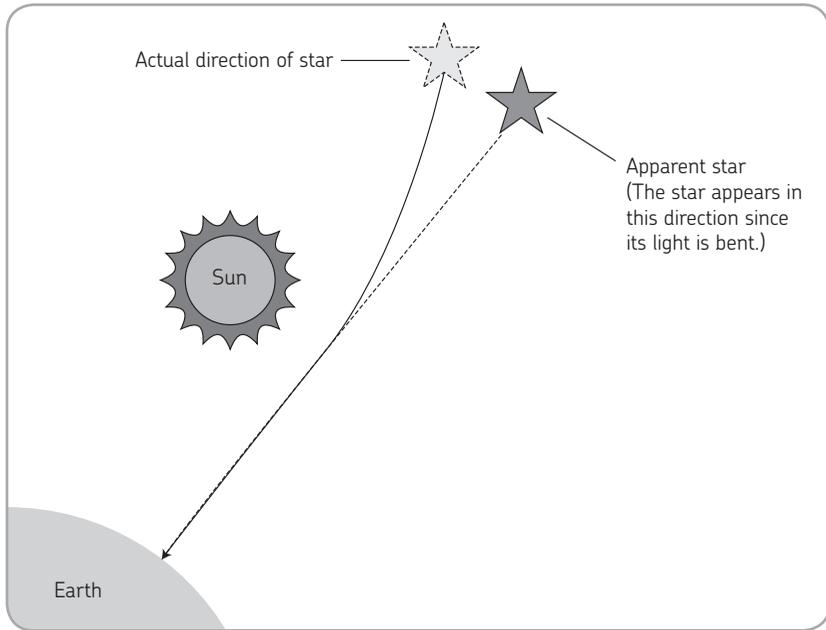


Figure 4-4: Bending of light near a large mass

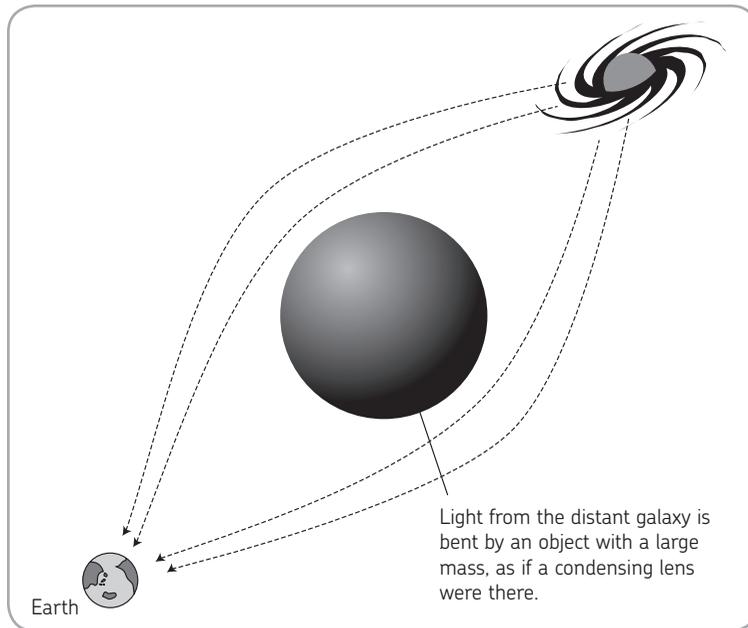


Figure 4-5: Gravitational lensing

## ANOMALOUS PERIHELION PRECESSION OF MERCURY

The *perihelion* is the point in the orbit of a planet that is closest to the Sun, as shown in Figure 4-6. We know that the perihelion of Mercury moves by approximately 574 arc seconds per century. Note that the “seconds” mentioned here are angular units rather than units of time. A minute of an arc is 1/60 of 1 degree, and a second is 1/60 of that. In other words, an arc second is 1/3600 of a degree. If the shift revolves by approximately 547 arc seconds per century, it is a shift of only approximately 0.16 degree per century.

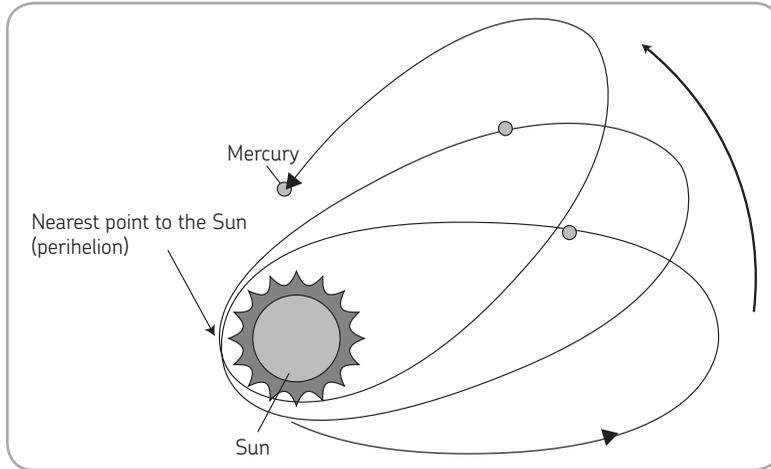


Figure 4-6: The anomalous shift in the perihelion of Mercury

Various causes of this shift, such as the effects of gravity of other planets, were investigated using Newtonian mechanics. However, none of these investigations were able to account for 43 arc seconds of perihelion movement. But when general relativity was used to check the shift in the perihelion of Mercury by calculating the Sun's warping of space-time, it was found to be shifted by exactly 43 arc seconds.

## BLACK HOLES

A *black hole* is a condition in which mass is extremely concentrated and gravity becomes so strong that not even light can escape from it.

A supernova stellar explosion occurs at the end of the life of a star having a mass several times that of the Sun. This event forms a region in space in which mass is extremely concentrated and gravity becomes stronger. Gravity becomes so strong that even light may not be able to escape. That region is a black hole.

Since light cannot escape from it, a black hole cannot be directly observed. However, if other stars exist in the vicinity of the black hole, gas from those stars will stream toward the black hole and form an accretion disk, that is, a cloud of diffuse matter around the black hole. When gas streams into the black hole from that accretion disk, X-rays and gamma rays are emitted.

A black hole candidate was discovered in 1971 in the constellation Cygnus, and currently, supermassive black holes are believed to exist at the center of galactic systems.

## GLOBAL POSITIONING SYSTEM AND RELATIVITY

The Global Positioning System (GPS) uses 24 satellites orbiting the Earth to determine position. Each satellite broadcasts a signal toward Earth that includes the radio broadcast time. A receiver on the ground (such as a car navigation system) receives those signals. The radio waves of the signals reach the receiver at the speed of light (approximately 300,000,000 m/s).

When the time the signal was received is compared with the broadcast time, and that time difference is multiplied by the speed of light, the distance to the satellite is known. In other words, if we assume that the distance between the satellite and receiver is 20,000 km, then the radio waves reach the receiver in  $20,000,000 \text{ m} \div 300,000,000 \text{ m/s} = 0.067$  seconds. That calculation is performed using radio waves from three satellites to accurately determine the position on the ground.

However, if there is an error in that time difference, an error will also occur in the calculation of the distance between the satellite and receiver. For example, if the satellite broadcast time is offset by 1 microsecond ( $10^{-6}$  second), the distance will be offset by 300 meters ( $300,000,000 \text{ m/s} \times 0.000001 \text{ s} = 300 \text{ m}$ ).

A GPS satellite orbits the Earth at an altitude of 20,000 km and a velocity that causes it to make 1 revolution in approximately 12 hours. At that velocity, the effect of special relativity causes its time to slow by 7.1 microseconds per day. However, since it is located high above the surface of the Earth, the effect of general relativity causes its time to pass faster than time on the surface of the Earth by 46.3 microseconds per day (the effect of general relativity is represented by equation 7 on page 161). As a result, the time broadcast from the GPS is slowed by 39.2 microseconds per day (see Figure 4-7). The design of the GPS system takes into consideration the effects of both special and general relativity with extreme precision.

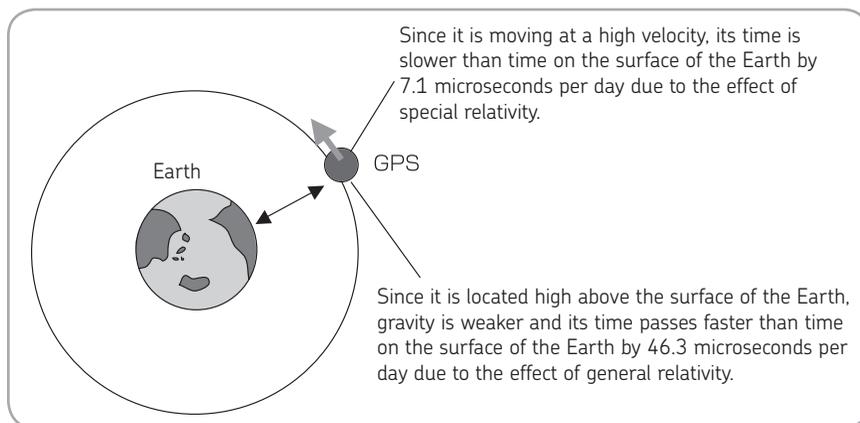
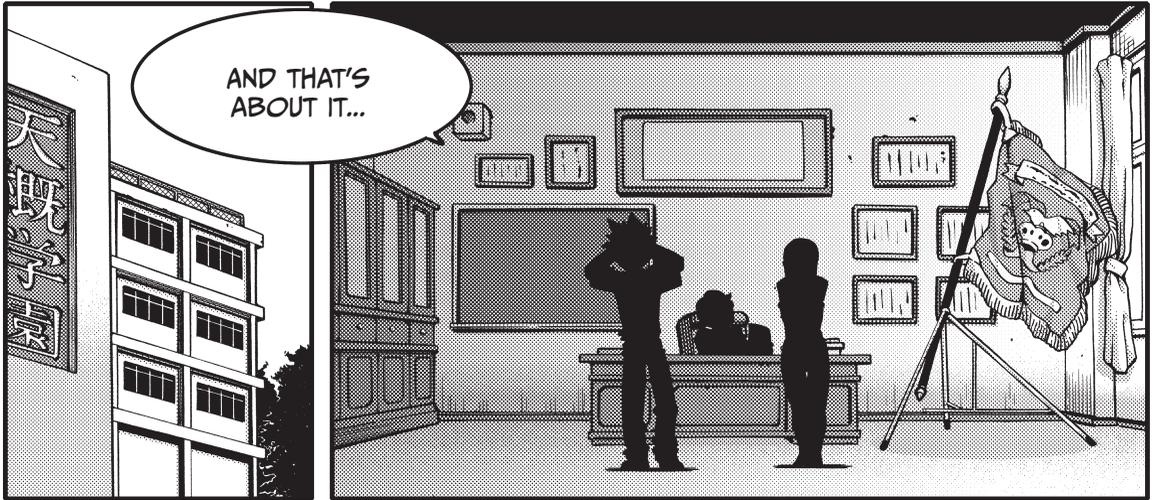


Figure 4-7: The Global Positioning System compensates for the effects of relativity.









YOU...YOU ARE...  
THE SCHOOL  
SUPERINTENDENT?!

YES.

THIS SCHOOL  
HAS A  
SUPERINTENDENT?!

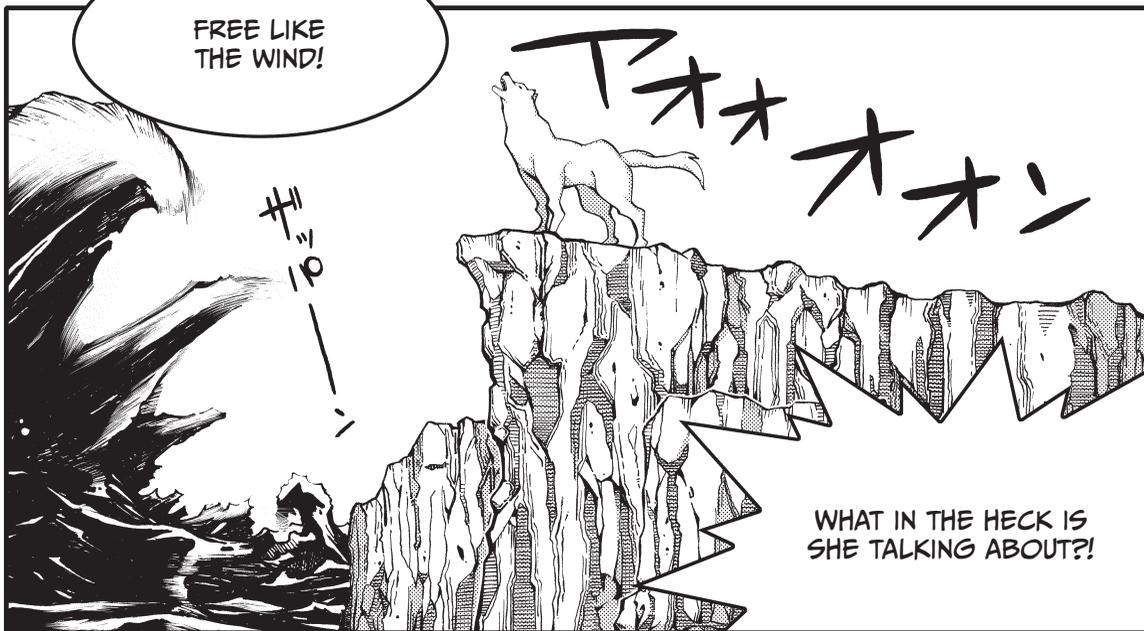


I DISGUISED MYSELF  
AS A DOG TO OBSERVE  
EVERYDAY SCHOOL LIFE  
IN SECRET.

EVEN THOUGH I  
HADN'T ACHIEVED  
ANYTHING BEFORE  
THEN...

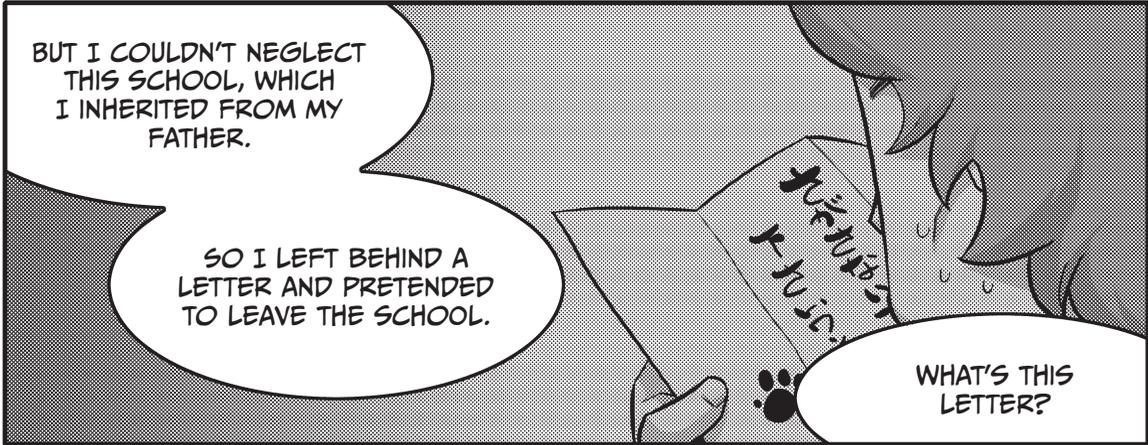


I INHERITED THIS  
POSITION FROM MY  
FATHER WHO PASSED  
AWAY, BUT I WANTED  
TO BE FREE!



FREE LIKE  
THE WIND!

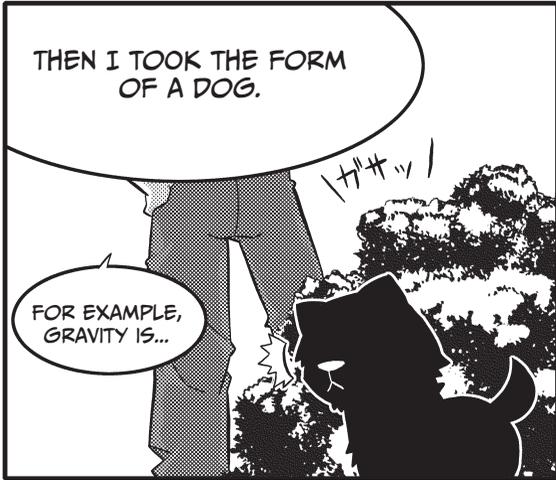
WHAT IN THE HECK IS  
SHE TALKING ABOUT?!



BUT I COULDN'T NEGLECT THIS SCHOOL, WHICH I INHERITED FROM MY FATHER.

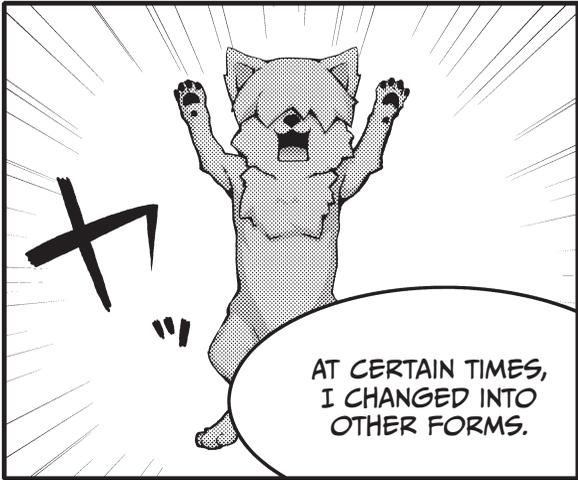
SO I LEFT BEHIND A LETTER AND PRETENDED TO LEAVE THE SCHOOL.

WHAT'S THIS LETTER?

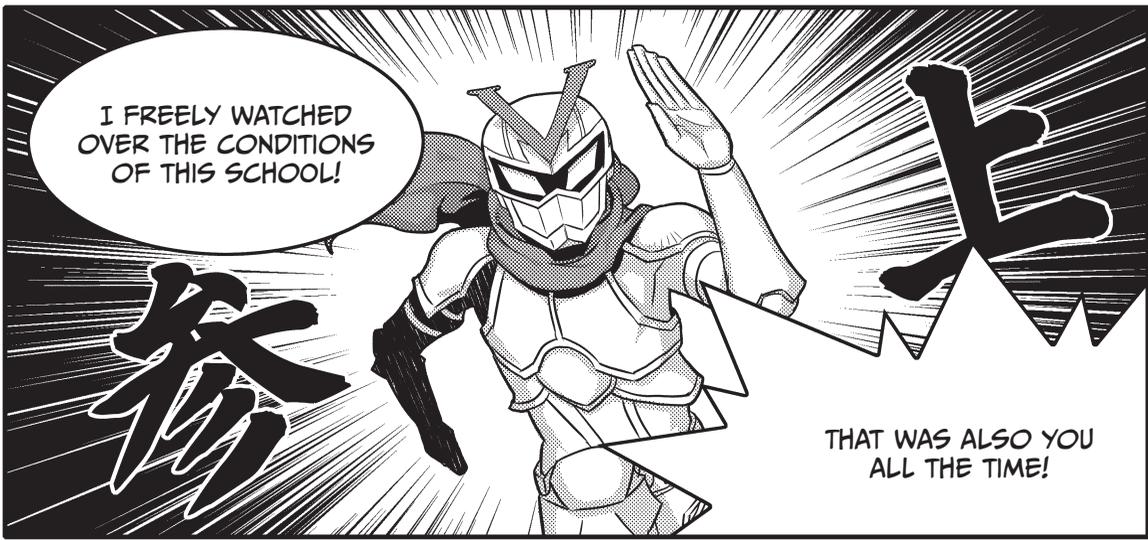


THEN I TOOK THE FORM OF A DOG.

FOR EXAMPLE, GRAVITY IS...

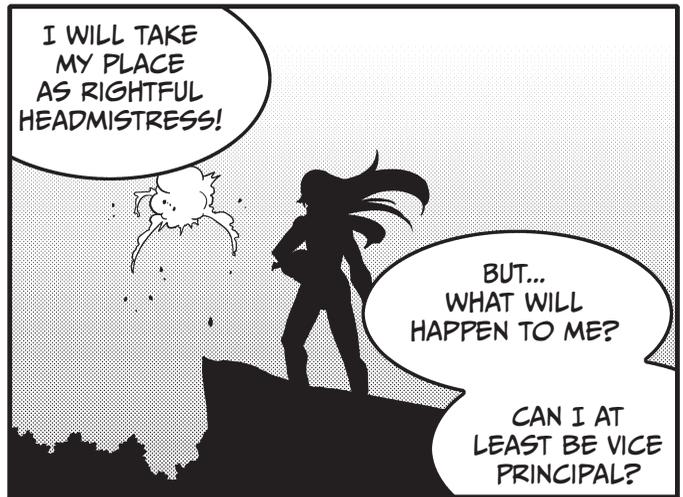
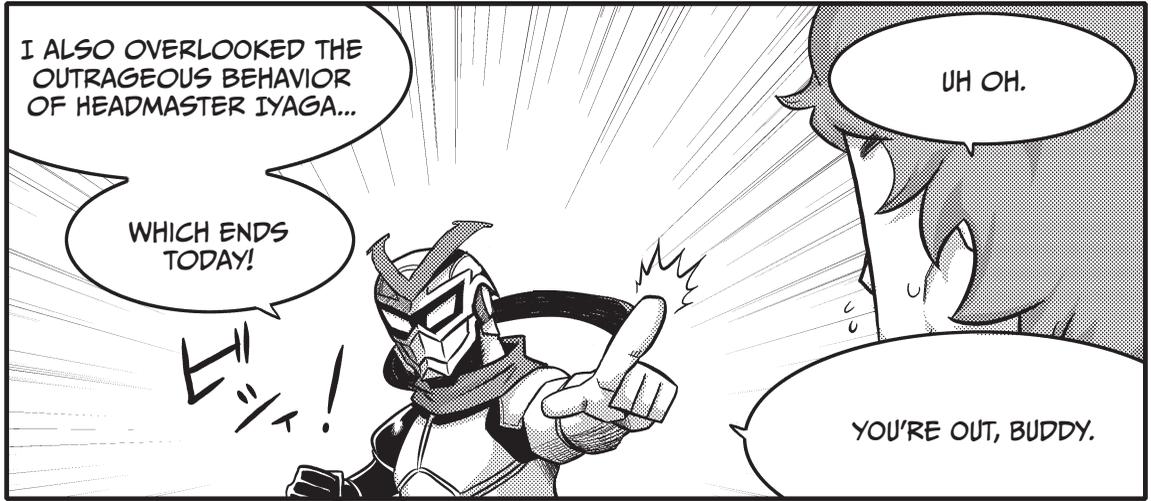


AT CERTAIN TIMES, I CHANGED INTO OTHER FORMS.



I FREELY WATCHED OVER THE CONDITIONS OF THIS SCHOOL!

THAT WAS ALSO YOU ALL THE TIME!



FOR VICE PRINCIPAL,

MISS URAGA! WILL YOU PLEASE ACCEPT THIS POSITION?

SHOCK

HUH?!

LITTLE OLD ME FOR SUCH AN ESSENTIAL... POWERFUL... POSITION?

SHE'S ACTING MODEST, BUT SHE SURE HAS A BIG SMILE ON HER FACE!

I WANT YOU TO STICK TO THAT STUDENT-CENTERED ATTITUDE!

YES, OKAY! I'LL DO THE BEST I CAN WITH MY LIMITED ABILITIES!

THIS IS NOT A STUDENT-CENTERED ATTITUDE! LOOK MORE CLOSELY!

MORE LIKE SUPERHUMAN ABILITIES! OUCH OUCH OUCH!

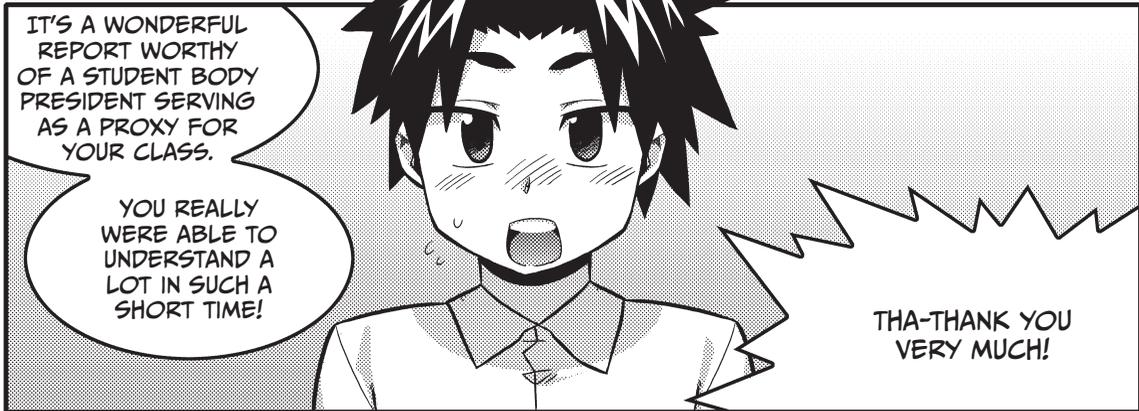
AND...MINAGI!

OWWWW...

YES?

I WAS VERY IMPRESSED BY YOUR REPORT!

REALLY?



IT'S A WONDERFUL REPORT WORTHY OF A STUDENT BODY PRESIDENT SERVING AS A PROXY FOR YOUR CLASS.

YOU REALLY WERE ABLE TO UNDERSTAND A LOT IN SUCH A SHORT TIME!

THA-THANK YOU VERY MUCH!



WELL, NOW THAT WE'RE ALL FRIENDS...



WHAT... ARE YOU SHY?



THAT'S NONE OF YOUR BUSINESS... TEACHER!



SWEEP SWEEP



WE SHOULD ENJOY OUR LAST DAY OF SUMMER VACATION— LET'S GO TO THE BEACH!

あー ははは

THIS SCHOOL IS SO WEIRD...



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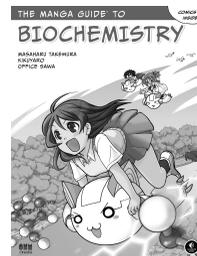
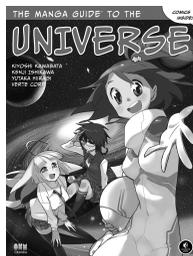
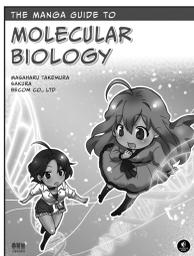
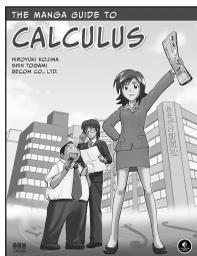
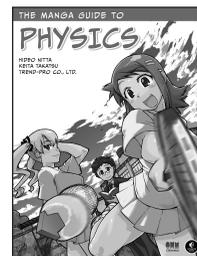
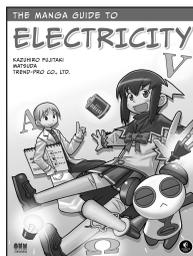
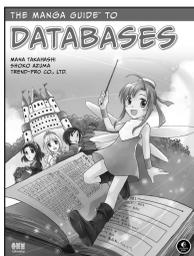
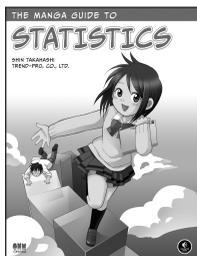
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## A CHARMING CARTOON GUIDE TO THE THEORY OF RELATIVITY

EVERYTHING'S GONE SCREWY AT TAKAI ACADEMY. WHEN THE CRAZY HEADMASTER FORCES MINAGI'S ENTIRE CLASS TO STUDY EINSTEIN'S THEORY OF RELATIVITY OVER SUMMER SCHOOL, MINAGI VOLUNTEERS TO GO IN THEIR PLACE. THERE'S JUST ONE PROBLEM: HE'S NEVER EVEN HEARD OF RELATIVITY BEFORE! LUCKILY, MINAGI HAS THE PLUCKY MISS URAGA TO TEACH HIM.

FOLLOW ALONG WITH *THE MANGA GUIDE TO RELATIVITY* AS MINAGI LEARNS ABOUT THE NON-INTUITIVE LAWS THAT SHAPE OUR UNIVERSE. BEFORE YOU KNOW IT, YOU'LL MASTER DIFFICULT CONCEPTS LIKE INERTIAL FRAMES OF REFERENCE, UNIFIED SPACE-TIME, AND THE EQUIVALENCE PRINCIPLE. YOU'LL EVEN SEE HOW RELATIVITY AFFECTS MODERN ASTRONOMY AND DISCOVER WHY GPS SYSTEMS AND OTHER EVERYDAY TECHNOLOGIES DEPEND ON EINSTEIN'S EXTRAORDINARY DISCOVERY.

*THE MANGA GUIDE TO RELATIVITY* ALSO TEACHES YOU HOW TO:

- ✓ UNDERSTAND AND USE  $E = mc^2$ , THE WORLD'S MOST FAMOUS EQUATION
- ✓ CALCULATE THE EFFECTS OF TIME DILATION USING THE PYTHAGOREAN THEOREM
- ✓ UNDERSTAND CLASSIC THOUGHT EXPERIMENTS LIKE THE TWIN PARADOX, AND SEE WHY LENGTH CONTRACTS AND MASS INCREASES AT RELATIVISTIC SPEEDS
- ✓ GRASP THE UNDERPINNINGS OF EINSTEIN'S SPECIAL AND GENERAL THEORIES OF RELATIVITY

IF THE IDEA OF BENDING SPACE AND TIME REALLY WARPS YOUR BRAIN, *THE MANGA GUIDE TO RELATIVITY* WILL STRAIGHTEN THINGS OUT.



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